

# A macrofounded linear stochastic discount factor

## An application to foreign exchange reserves asset allocation

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# Motivation

Central banks as mechanism to deal with sudden stop risk

- Sudden stop causes:
  - Eichengreen, Hausmann & Panizza (2002) on the original sin;
  - Caballero and Krishnamurthy (2003) on low financial development;
- The role of reserves:
  - Caballero & Krishnamurth (2004) on reserves as a precautionary savings mechanism;
  - Caballero & Panageas (2008) on reserves as a hedging mechanism;

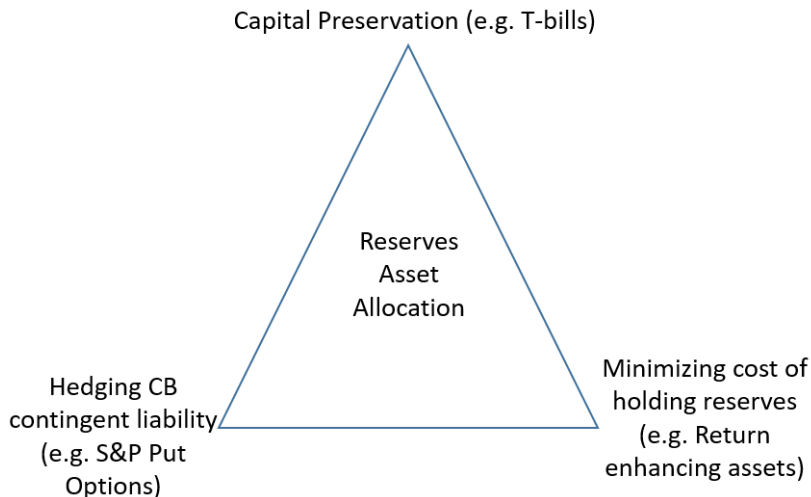
# Motivation

## Societies' mandate with the Central Bank

- In this paper I calibrate a simple macro-finance model that can guide us on understanding:
  - A linear macrofounded stochastic discount factor that a Central Bank can use to take reserves' portfolio choice decisions;
- I propose a reserves' asset allocation trinity that encompass three objectives:
  - Capital preservation;
  - Sudden stop hedging risk;
  - Return enhancing;

# What is this paper is about?

Reserves' asset allocation trinity



# What is this paper is about?

## Solving the reserves' asset allocation trinity

- The problem of a benevolent Central Bank at deciding reserves asset allocation:

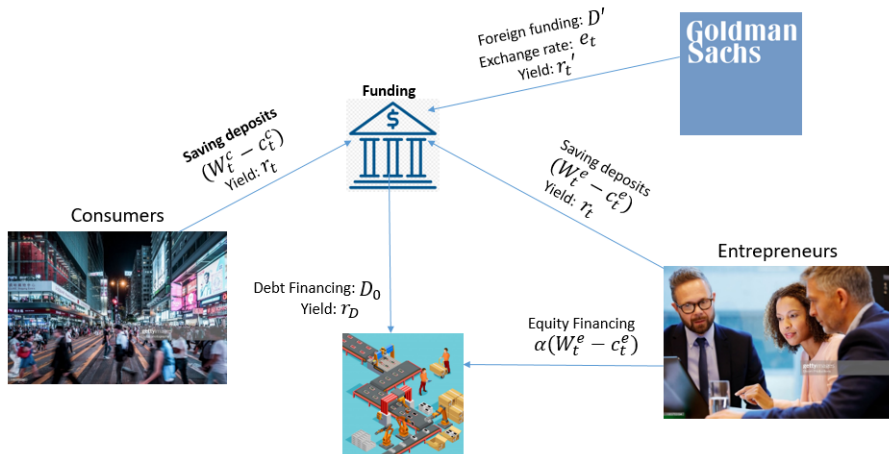
$$\underset{w}{\text{maximize}} \quad E[w^T r_t - r_h] - \frac{1}{2} \gamma_v \text{Var}[w^T r_t] - \frac{1}{2} \gamma_{te} \text{Var}[w^T \beta f_t - \beta_l f_t]$$

$$\text{subject to} \quad 0 \leq w_i \leq 1, \quad i = 1, \dots, N.$$

- $w$ : portfolio weights;
- $r_t$ : investable asset returns;
- $r_h$ : social cost of holding reserves;
- $\gamma_v$ : capital preservation preference;
- $\gamma_{te}$ : Sudden stop hedging motive;
- $\beta$ : Assets' factor loadings;
- $f_t$ : Relevant risk factors (**CB's stochastic discount factor**);
- $\beta_l$ : Liability factor loadings;

# Macro Finance Model

## Local Economy



# Macro Finance Model

## Exogenous macro variables

- Real aggregate returns on investments:

$$g_t = \bar{g} + \epsilon_{g,t}$$

- Inflation rate:

$$\pi_t = \bar{\pi} + \beta_{\pi,g}g_t + \beta_{\pi,er}er_t + \epsilon_{\pi,t}$$

- Nominal exchange rate changes:

$$er_t = \beta_r(r_{t-1} - r_{i,t-1}) + \epsilon_{er,t}$$

- Foreign interest rates:

$$r_{i,t} = \bar{r}_i + \beta_i r_{i,t-1} + \epsilon_{ri,t}$$

- Potentially correlated shocks  $\Sigma_\epsilon$ .

- Three period problem:
  - Consumers decide consumption ( $C_t^c$ ) accordingly with CRRA preferences with risk aversion  $\gamma_c$  and endowment  $W_0^c$ ;
  - Entrepreneurs decide consumption ( $C_t^e$ ), investment ( $\alpha$ ), and leverage ( $D_0$ ) accordingly with CRRA utility function with risk aversion  $\gamma_e$  and endowment  $W_0^e$ ;
  - Returns on entrepreneurs investment are only available at the last period;
  - The bank ex-ante fix cost of debt ( $r_D$ ) to break-even, on average, requiring a premium for being risk averse (exponential utility);
  - Deposits are offered in perfectly elastic supply, and rates are set to compensate consumers' exposure to inflation;



# Macro Finance Model

## A sudden stop

Sudden Stop as a jump on exchange rate volatility:  $\uparrow \sigma_e$

- Lower consumption:
  - Higher inflation risk



- Lower investment:
  - Higher cost of debt;
  - Higher equity risk premium;

Consumption



Investment

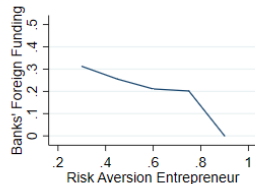
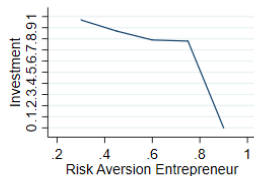
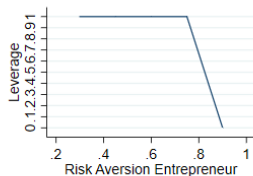
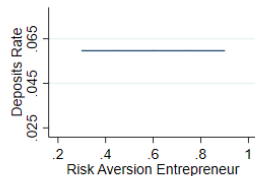
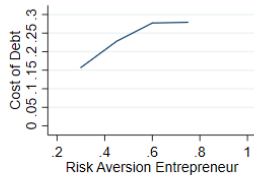
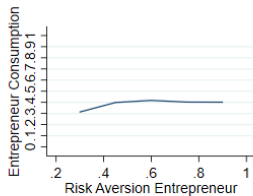
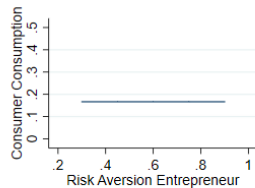


# Model Calibration

Chile (1990-2018)

- $\bar{g} = 4.62\%$  and  $\sigma_{\epsilon,g} = 2.7\%$
- $\bar{\pi} = -1.6\%$ ,  $\beta_{\pi,g} = 1.3$ ,  $\beta_{\pi,er} = 0.48$  and  $\sigma_{\epsilon,\pi} = 4.4\%$
- $\beta_r = 0.52$  and  $\sigma_{\epsilon,er} = 7.1\%$
- $\bar{r}_i = 0.28\%$ ,  $\beta_i = 0.87$  and  $\sigma_{\epsilon,ri} = 1.2\%$
- $\rho_{er,g}^{\epsilon} = -0.56$ ;
- $\rho_{ri,er}^{\epsilon} = -0.4$ ;
- $W_0^c = 0.25$ ;
- $W_0^e = 1.0$ ;
- $\gamma_c = 3.0$ ;
- $\gamma_e \ll \gamma_c$ ;
- $\gamma_b = 1$ ;

# Base case equilibrium



# Simulating a sudden stop inside the model

- Assume  $\gamma_e = 0.45$ ;
- A sudden stop is an increase of 30% in exchange rate risk (7%  $\rightarrow$  9.5%);
- Equilibrium changes:
  - Higher deposit rates;
  - Higher entrepreneurs' consumption;
  - Entrepreneurs maintain higher levels of liquidity;
  - Entrepreneurs maintain higher levels of liquidity, instead of investing in the risky project;
  - A sudden stop has a negative effect on social welfare, measured in aggregated certainty equivalents;

# Introducing a Central Bank

How can a Central Bank can intervene in this economy?

- CB takes money from consumers and entrepreneurs in normal times;
- CB commits to provide resources in “sudden stops sates of the world” that are collected from “good states of the world” to ;
- This resources are the reserves in this model;

Central Banks have a contingent liability:



Sudden Stop



# Introducing a Central Bank

How can a Central Bank can intervene in this economy?

- In this context the CB is selling an final option to society;
- For example, in this framework a CB intervenes the market when currency markets are affecting social welfare:
  - Investment opportunities;
  - Credit conditions;
  - Inflation risk;
  - Equity risk premium;
  - Liquidity risk;
- In this model, a benevolent CB has the following objectives:
  - Minimize the amount of resources taken from the public today;
  - Minimize the volatility of reserves;
  - Maximize the correlation of invested reserves and sudden stop risk;
- Social preferences implicitly determine the weight of each objective;

# Introducing a Central Bank

How can a Central Bank can intervene in this economy?

- The main practical lesson coming from this model is that CB's contingent liability depends on the macroeconomic equilibrium in different states of the world;
- I argue that the model presented in this can be approximated by a linear stochastic discount factor a la Chen, Roll & Ross (1986);
- CRR is a five linear factor model that includes:
  - Equity market risk ( $r_{M,t} - r_{f,t}$ );
  - GDP Growth Expectations ( $E[g_t]$ );
  - Inflation risk Expectations ( $E[\pi_t]$ );
  - Term premiums ( $r_{l,t} - r_{f,t}$ );
  - Credit premiums ( $r_{d,t} - r_{f,t}$ );

# Solving a Reserves' Asset Allocation Problem in Practice

The case of Chile

- The value changes in the contingent liability of CB are measured from 1m implied volatility of CLP-USD options (1998-2019);
- Historical returns on investable assets:
  - Gold;
  - Oil;
  - Global Bonds (JPM GBI);
  - EM Bonds (JPM EMBI);
  - Asia Pacific Equities;
  - EM Equities (MSCI EM);
  - All Countries Equities (MSCI ACWI);
  - DM Equities (MSCI World);
  - SP Put Option (VIX);



# Solving a Reserves' Asset Allocation Problem in Practice

## The case of Chile

- Estimation of CRR factor model:
  - Chilean equity market (IPSA) returns minus monthly returns of short term deposits (Riskamerica Intermediación Financiera);
  - GDP growth expectations from the Chilean Central Bank Survey;
  - Inflation expectations from the Chilean Central Bank Survey;
  - Return of long term government bonds (Riskamerica Gobierno Chile) minus monthly returns of short term deposits (Riskamerica Intermediación Financiera);
  - Return of corporate bonds (Riskamerica Corporativo Chile) minus monthly returns of long term government bonds (Riskamerica Gobierno Chile);

# Solving a Reserves' Asset Allocation Problem in Practice

The case of Chile

- International CAPM risk premium estimates:

Monthly Returns in USD	$E[r-r_f]$	Beta CI 5%	Beta	Beta CI 95%	Volatility	Sharpe Ratio
Gold	0.06%	0.06	0.16	0.38	4.91%	0.01
Oil	0.30%	0.46	0.79	1.12	9.08%	0.03
Global Bonds	-0.11%	-0.35	-0.28	-0.20	2.11%	-0.05
EM Bonds	0.13%	0.23	0.34	0.44	2.41%	0.05
Asia Pacific Equities	0.36%	0.87	0.94	1.01	4.63%	0.08
EM Equities	0.48%	1.15	1.23	1.32	6.02%	0.08
All Countries Equities	0.39%	1.00	1.00	1.00	4.35%	0.09
DM Equities	0.38%	0.96	0.97	0.98	4.24%	0.09
S&P Put Option	-1.38%	-4.30	-3.57	-2.84	22.88%	-0.06

# Solving a Reserves' Asset Allocation Problem in Practice

## The case of Chile

- Assets are spanned by the macro risk factors:\*

Monthly Returns in US	Equity Factor	Growth Factor	Inflation Factor	Credit Factor	Term Premium Factor
Gold	0.18*	0.12%	-0.16%	-0.03	0.78*
Oil	0.19	0.34%	-1.02%*	2.30*	-1.35*
Global Bonds	-0.11*	-0.16%*	0.19%*	-0.17	0.83*
EM Bonds	0.18*	-0.11%	-0.25%	0.45	0.13
Asia Pacific Equities	0.34*	0.04%	-0.92%*	0.38	-1.03*
EM Equities	0.56*	0.08%	-1.10%*	0.72	-0.94*
All Countries Equities	0.36*	0.09%	-0.79%*	0.42	-1.03*
DM Equities	0.33*	0.10%	-0.76%*	0.40	-1.03*
T-bills 3 mo	0.00	0.00%	0.00%	0.00	0.00
S&P Put Option	-1.62*	-0.51%	2.21%	-0.01	2.82*

\*The stars indicate an, at least, 10% statistical significance. 


# Solving a Reserves' Asset Allocation Problem in Practice

The case of Chile

- CB contingent liability is spanned by the macro risk factors:<sup>†</sup>

	<b>CI 5%</b>	$\beta$	<b>CI 95%</b>
Equity Factor	-1.43	-0.92	-0.41
Growth Factor	-0.02	-0.01	0.01
Inflation Factor	-0.01	0.01	0.02
Credit Factor	-8.61	-3.86	0.88
Term Premium Factor	-0.88	0.56	2.00

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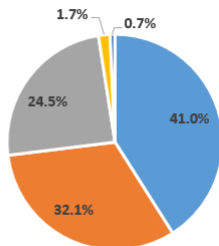
<sup>†</sup>Value changes in CB's contingent liability are measured in dollars. 

# Solving a Reserves' Asset Allocation Problem in Practice

## The case of Chile

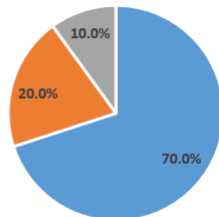
- $r_f = 1.54\%$  (T-bills)
- Reserves cost: UST 10Y 1.92% + Chile CDS 0.95% = 0.24% mo
- $\gamma_v = 3$
- $\gamma_{te} = 2$

Optimal Reserves Portfolio



■ T-bills 3 mo ■ Global Bonds ■ DM Equities ■ S&P Put Option ■ Others

Benchmark Portfolio



■ Global Bonds ■ T-bills 3 mo ■ Gold

# Solving a Reserves' Asset Allocation Problem in Practice

The case of Chile

- $r_f = 1.54\%$  (T-bills)
- Reserves cost: UST 10Y 1.92% + Chile CDS 0.95% = 0.24% mo
- $\gamma_v = 3$
- $\gamma_{te} = 2$

<b>Annualized Returns</b>	<b>Optimal Reserves Portfolio</b>	<b>Benchmark Portfolio</b>
Expected Return	1.92%	0.72%
Volatility	2.53%	5.30%
Tracking Error	24.18%	21.96%
Cost of Reserves		2.87%

# Conclusions

- The role of Central Banks as a social insurance mechanism has been well established in the international economics literature;
- In this paper I develop a macro finance model that links sudden stops with a well-recognized factor model of the empirical asset pricing literature, Chen, Roll & Ross (1986);
- Using this macrofounded stochastic discount factor, I solve the proposed reserves' asset allocation trinity from perspective of Chilean Central Bank;
- Based on the estimated parameters I find space for improving the efficiency of a typical reserves portfolio;

## **Comments to: “A Macrofounded Linear Stochastic Discount Factor: An Application to Foreign Exchange Reserves Asset Allocation”**

by Jorge Sabat

Presented by:  
Marco Ortiz

2020 First Conference on Financial Stability  
and Sustainability

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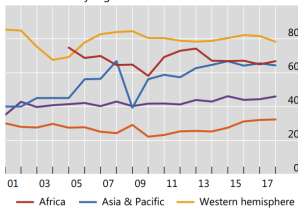




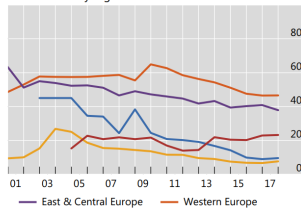
## Overview

- ▶ Contribution: Provide a normative model of strategic asset allocation for a central bank holding reserves that supports different considerations for holding foreign reserves.
  - ▶ The main idea is to make central banks shift from a dollar allocation to a risk allocation
  - ▶ The median for the proportion of assets invested in U.S. dollar-denominated securities is 68 percent across 99 central banks.

Share of USD by region



Share of EUR by region



## Overview

- ▶ *Comments*

- ▶ Very interesting paper on a very relevant topic.

- ▶ International reserves accumulation is still a puzzle for economists.

- ▶ Financial crisis highlighted the importance of quantitative tools as part of the macropru toolkit.

- ▶ Weighing costs and benefits is still a obscure subject with many potential avenues to follow.

- ▶ This paper:

- ▶ Regardless the *why*... here is a menu for the *how*.

## *Literature Background*

- ▶ Plenty of avenues:
  - ▶ *Macropu/precautionary motives*: Aizenman & Marin Lee (2005); Aizenman, Chinn & Ito (2014), Benigno and Fornaro (2012); Bigio & Bianchi (forthcoming); Bianchi (2014).
  - ▶ *FX intervention*: Chang, R (2018); Basu et al (2016); Gabaix and Maggiori (2015); Blanchard et al. (2015); Cavallino (2019).
  - ▶ *Reserves Adequacy*: Heller (1966); IMF (2011); Jeanne (2007); Jeanne & Ranciere (2011); Ruiz-Arranz and Zavadjil (2008).
  - ▶ *Optimal portfolio*: Eichengreen (2005), Zhang et al. (2013); Aizenman & Glick (2009); García-Pulgarín et al. (2015); Papaioannou et al. (2006).

## Sabat's Formulation

- ▶ Author identifies three objectives: (i) Minimize 'yield give up'; (ii) Provide conditional foreign currency liquidity; (iii) Capital preservation (yield).

**Motives for holding foreign exchange reserves**

Motives	Highly relevant	Somewhat relevant	Not relevant
<u>Provide self-insurance against potential external shocks</u>	84%	10%	1%
<u>Conduct foreign exchange policy</u>	66%	20%	12%
Service external debt or obligations	55%	21%	16%
Support monetary policy operations	27%	46%	17%
<u>Ensure savings for intergenerational equity</u>	9%	26%	52%
Other	4%	5%	5%

N = 99

Source: RAMP Survey on the Reserve Management Practices of Central Banks.

## *Sabat's Formulation*

- ▶ The author's approach involves:
  - ▶ Construct a series of factor models.
  - ▶ Formulate an asset liability optimization model (mean variance optimizer).
  - ▶ Construct liabilities using a mix of debt and contingent liabilities (forex liquidity provision and financial sector solvency).
- ▶ With that, it is possible to calculate the optimal exposure to risk factors (return and hedging motives).
- ▶ Author takes into account potential restrictions to asset classes.
- ▶ Finally adds a capital preservation motive.
- ▶ Model is tested for Chile, a small open economy with a high exposure to copper prices.

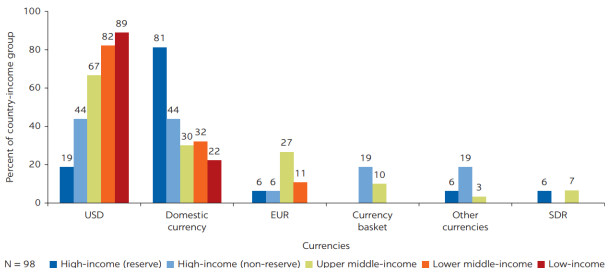
## *Comments*

- ▶ What about risk premia and the cost of funding?
- ▶ Governance and transparency?
- ▶ How well does the alternative portfolio do in out of sample exercises.
- ▶ All crises are different, can it be tested for the GFC? Is the sample (1989-2016) relevant?
- ▶ How volatile are the factors weights for sample changes?
- ▶ Should we consider all banks the same (MSCI Chile Banks)?
- ▶ Can we estimate foreign liquidity provision episodes to the data?

## Comments

- ▶ The “why” is a puzzle in itself. Why reserves composition are so tightly linked to trade and capital flows? Maybe it has something to say about the “how” that we are missing.

Nominaire choice by country-income category



Source: RAMP Survey on the Reserve Management Practices of Central Banks.

## *Final Comments*

- ▶ Extremely interesting approach to a more and more relevant subject.
- ▶ A practitioner approach that raises questions about why central banks behave the way they behave.
- ▶ Still plenty of questions to explore that might affect policy recommendations.



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