A macrofounded linear stochastic discount factor: An application to foreign exchange reserves asset allocation

By Jorge Sabat[†]

(Aizenman and Glick, 2009) illustrates the social role of Central Banks in conducting sudden stop risk-management. This policy decision implies that a Central Bank holds a "short position" on a complex put option on the exchange rate between the local and the reserve currency. The main policy tool to hedge this contingent liability is the reserves portfolio. In this paper I argue that because of macroeconomic stability considerations, a Central Bank can solve the portfolio problem using a linear stochastic discount factor that follows Chen et al. (1986) linear factor model. This is, a Central Bank focus its attention on the effect of sudden stops on GDP growth and inflation expectations, the local equity risk premium, as well as, credit and liquidity risk. In this paper I solve a normative model of asset-liability management based on these five risk factors. I implement the model for the case of Chile, a small and open economy exposed to commodities.

I. Introduction

There are two theoretical arguments that justify that a Central Bank holds reserves in a floating exchange rate economy. First, a mercantilist motive, that imply keeping the exchange rate undervalued to promote exports. Second, precautionary motives, where reserves work as self-insurance against sudden-stop risk.[1] In this paper, I mainly focus on the precautionary channel. I propose a normative model of strategic asset allocation for a Central Bank. A decision problem that arise from countries holding reserves for macroeconomic stability considerations related to exchange rate misalignment's correction, provision of liquidity during sudden stops, as well as, the reductions in the probability of neg-

[†] Finance Department, Universidad Diego Portales

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[1]A long list of papers analyze these two objectives. On the mercantilist side. (Dooley et al., 2003) focus on export-oriented policies. (Rajan and Subramanian, 2011) focus on "Dutch diseases". (Daude et al., 2016) see reserves as a "corrective tool". (Rodrik, 2006; Aizenman and Marion, 2004; Aizenman and Lee, 2005, 2007; Bianchi et al., 2012; Jeanne, 2007; Jeanne and Ranciere, 2011; Benigno and Fornaro, 2012) make the case in favor of reserves as a self-insurance against sudden-stops.

ative economic shocks at country level.[2] As a consequence of country's reserve policy, Central Banks hold a short position on a put option on the exchange rate between the local and the reserve currency. Consequently, Central Banks decide how to allocate reserves across different asset classes focusing on a set of relevant macroeconomic risk factors, in order to hedge this contingent liability. I argue that because of macroeconomic stability considerations, a Central Bank act as if they believe that representative agent's stochastic discount factor could be model as a Chen et al. (1986) linear factor model. This is equivalent to assume that the relevant state variables in an intertemporal precautionary savings model a la Merton et al. (1973) are approximated by the following linear risk factors: GDP growth and inflation expectations, the local equity risk premium, as well as, credit and liquidity risk.

In the literature only a few studies have tried to analyze the asset allocation problem of Central Banks reserves. (Eichengreen, 2005) analyze the trade-off between reserves diversification and the liquidity risk embebed in the reserves portfolio. (Papaioannou et al., 2006) analyzes the problem using a mean-variance optimization framework with liquidity costs to estimate optimal portfolio weights among the main international currencies. (Zhang et al., 2013) solves a portfolio choice problem, considering a conditional value-at-risk minimization, and disappointment avoidance utility maximization. (Aizenman and Glick, 2009) study the asset allocation problem of a Central Bank that invests its reserves in order to minimize the probability of a sudden stop. (García-Pulgarín et al., 2015) solve the asset allocation problem of a Central Bank, separating the Central Bank objective in two sub-problems. First, a Safety Tranch, comprised of liquid, almost default-free and low volatile assets. Second, a Wealth Tranche, that aims to maximize the return with a broader range in the asset space and a longer investment horizon.

The main contribution of this paper is to propose a normative model that explicitly includes the macroeconomic stability considerations of a country. The results presented in this paper support the idea that Central Banks and Sovereign should shift from dollar allocations to risk allocations, using (Ilmanen and Kizer, 2012) words. As we will see, the model is flexible enough to encompass Central Banks' multi-objectives: minimization of the cost of holding reserves, hedging of its contingent liability, and its capital preservation mandate. In addition to a given set of exogenous constraints on investable assets. I implement the model using the case of Chile, a small an open economy with commodity exposure, documenting potential problems in the implementation, as well as, a quantitative evaluation of the trade-offs of including derivatives in the investable asset spectrum.

The rest of the paper is organized as follows. Section 2 describes the methodology and the model, and explains how assets and liabilities can be analyzed using

[2]It's not as clear how important are the social costs of holding reserves. (Rodrik, 2006) shows that the cost is around 1 percentage point of GDP annually for developing nations. Similarly, (Calvo et al., 1991), and (Filardo and Grenville, 2012), show that sterilized interventions are costly.

factor models. In Section 3, the application to the case of Chile is presented. Finally, Section 4 concludes.

II. Methodology

A. Factor Model

We start from a set of n-asset classes (fixed income, equities and currencies) that are traded in the market. In our framework, we will assume that assets returns follow an approximate factor structure, consistent with (Chamberlain and Rothschild, 1982) and (Ingersoll, 1984), the generalization of the classical arbitrage pricing theory developed by (Ross, 1976). This assumption will be important for our empirical application, since idiosyncratic components of returns do not need to be uncorrelated. In an approximate factor model, asset classes returns are given by:

$$r_t^n = c^n + B^n \widetilde{f}_t + \epsilon_t^n \tag{1}$$

Here c^n denotes a n-vector of constants, $\widetilde{f}$ the k-vector of systematic risk factors, B^n a $n \times k$-matrix of factor betas, and $\widetilde{\epsilon}_t^n$ a n-vector of idiosyncratic returns. As we mentioned above, one important characteristic of an approximate factor model is that the covariance matrix of idiosyncratic returns does not need to be diagonal.

The literature recognizes three types of factor models: i) Macroeconomic factors, based on observable economic and financial time series; ii) Fundamental factors, created from observerable asset characteristics; iii) Statistical factors, that are unobservable and are extracted directly from asset returns. While the empirical asset pricing literature has dedicated significant effort searching for factors, see for example (Cochrane, 2011) and (Harvey et al., 2016). There is no one accepted factor model able to explain the cross-sectional variation of a large variety of different asset classes. As a consequence, I propose different approaches to address that problem. Firstly, we can consider the global version of two traditional factor models, that are consider baseline models in the asset pricing literature. The Fama-French 3 Factor Model, (Fama and French, 2012), and the Fama-French 5 factor model, (Fama and French, 2017). Secondly, I use a macroeconomic founded model that is an adaptation of the traditional (Chen et al., 1986). Thirdly, I analyze other factors that are used by practitioners or that are statistically related to the variation of the market value of Central Banks' liabilities. Specifically, I propose a factor model based on the three-factor specification that maximizes the adjusted R-square of the time series regression of liabilities.

B. Asset Liability Management

Following Bodie and Brire (2014) we formulate the asset liability optimization model as follows:

$$\underset{w_f^*}{\text{maximize}} \quad E[r_f - r_l] + \frac{1}{2}(1-\rho)Var[r_f - r_l] \tag{2}$$

where r_f is the return of the asset portfolio, r_l is the return of portfolio of liabilities, and $\rho > 0$ is the risk aversion coefficient. Two assumptions are implicit in this specification. First, Central Banks are mean-variance optimizers, and consequently will not hedge unanticipated shocks to time varying investment opportunities. In practice, this assumption can be defended since, for a wide variety of preferences, hedging demands for risky assets are typically small, even nonexistent as (Ait-Sahalia and Brandt, 2008) and (Brandt, 2009) have shown. Second, I assume that Central Bank optimal portfolio choice is not influenced by Central Banks solvency or leverage.[3]

In this case, we can solve the unconstrained optimal factor based portfolio analytically:

$$w_f^* = \frac{\mu_f}{(\rho-1)\Omega_f} + \frac{\Omega_{f_l}}{\Omega_f} \tag{3}$$

where μ is a vector with factors' expected risk premiums, Ω_f is the variance-covariance matrix of the risk factors, and Ω is a vector that contains the covariance between the factors and the liabilities.

C. Central Bank Liabilities

The liabilities that Central Banks hedge are intimately related with the institutional framework of the country. For example, the IMF (2014) states that, in order to identify Central Banks' liabilities is important to consider the amount of debt issued, sovereign credit risk, interest rate duration and exchange rate risk, as well as, contingent liabilities which dynamics (and size) are usually unobservable. Specifically, we focus our attention in explicit debt, debt issued by the Central Bank, and contingent liabilities related to the provision of foreign exchange liquidity and financial sector solvency. While the size and the cost of Central Bank's debt can be measured directly from traded bonds or indirectly from bonds issued by the Central Government, the market value of contingent liabilities have to be estimated. Specifically, we focus in two contingent liabilities, financial bailouts, which are related to the explicit or implicit deposit insurance. Emergency liquidity provision in foreign currency, which is related to the access

[3] (Hall and Reis, 2015) analyze financial stability of central banks. The literature tends to acknowledge that capitalization of Central Banks could affect monetary policy decisions or independence from central governments.

to foreign capital markets during periods of financial stress. The cost of financial bailouts is estimated following the methodology of (Ronn and Verma, 1986). The main idea is that banks' assets follow a stochastic process with a volatility that can be estimated from banks' stock volatility. The liability is calculated as a put option on banks' assets with a strike price equal to banks' debt. In a Black-Scholes framework, the equity value of a bank will be equal to a call option on assets with strike price equal to the market value of debt. In this context, (Ronn and Verma, 1986) derive a closed form solution for the equity value of a bank. As the market value of assets is unobservable, the volatility of the market value of assets is estimated from the following set of equations.

$$E = VN(x) - \rho BN(x - \sigma_V \sqrt{T})$$
$$x = \frac{ln\left(\frac{V}{\rho B}\right) + \sigma 2T/2}{\sigma_V \sqrt{T}}$$
$$\sigma_V = \frac{\sigma_E E}{VN(x)} \tag{4}$$

where ρ is a measure of the relevant threshold of debt that will capture bank's default[4]; σ_V is the volatility of banks' assets; T is the option maturity; V and E are the value of assets and equity respectively; $N(\cdot)$ is the cummulative normal distribution function.

Finally, abstracting from the dividend payments made by banks, the cost of guaranteeing \$1 for the Central Bank will be given by:

$$y = \frac{ln(B/V) - \sigma_V^2 T/2}{\sigma_V \sqrt{T}} \tag{5}$$

On the other hand, the contingent liability related with the provision of foreign exchange liquidity is measured as a payer swaption on the spread of foreign currency borrowing. The main idea is that Central Banks provide liquidity at the 'long-run' spread, therefore this is liability is equal to a derivative which is in the money when high short term cost of borrowing is high. Modelling the swap rate as the spread on foreign currency using a Vasicek model, we could estimate the value of payer swaption with a fixed swap rate following (Hiibnerl, 1997). The closed form solution of a payer swaption under a Vasicek model will be given by:

[4](Ronn and Verma, 1986) calibrate this number in 0.97.

$$
\begin{aligned}
\Pi = P(r,t,T)N(-d_2) - \exp\Big(r_x(s-t)\Big)P(r,t,s)N(-d1) \\
\sigma_P = v(t,T)\frac{1-\exp\left(-a(s-t)\right)}{a} \\
d_1 = ln\Big(\frac{P(r,t,s)\exp\left(r_x(s-t)\right)}{P(r,t,T)}\Big) + \sigma_P/2 \\
d_2 = d_1 - \sigma_P \\
v^2(t,T) = \sigma_r 2(1-\exp -2a(T-t))/(2a) \\
P(r,t,T) = A(t,T)\exp -B(t,T)r \\
B(t,T) = \frac{1-\exp -a(T-t)}{a} \\
A(t,T) = \exp\frac{(B(t,T)-T+t)(a^2\bar{r}-\sigma_r^2/2}{a^2} - \frac{\sigma^2 B(t,T)^2}{4a}
\end{aligned} \tag{6}
$$

Alternatively, we can take an empirical approach to estimate the risk exposure of contingent liabilities. First, the cost of financial bailout can be estimated indirectly from the absolute value of idiosyncratic returns of the financial sector with respect to the local stock market, taking only the returns below the 5th percentile. Second, the contingent liability related to the provision of liquidity in foreign currency, could be estimated as the absolute value of returns of corporate bonds in foreign currency, taking only the returns below the 5th percentile It's worth noting that, is assumed that financial solvency's liability returns are measured in local currency, while liquidity's liability is in foreign currency.

D. Porfolio Replication

The optimal exposure to risk factors presented above, balances the return maximization (or yield give-up minimization) and liability hedging objectives of a Central Bank with a coefficient of risk aversion ρ. Nevertheless, a Central Bank usually is restricted to a subset of asset classes that are defined exogenously (e.g. constitutional amendment). Given a m-subset of investable assets, the Central Bank will replicate the optimal factor based portfolio minimizing the weighted difference exposure to systematic risks, as follow:

$$
\underset{w_a^*}{\text{minimize}} \quad [w_f^* - w_a^* B^m] W [w_f^* - w_a^* B^m]^T \quad \text{subject to} \quad w_a^* \geq 0, w_a^* \leq 1 \tag{7}
$$

Where w_a^* is the portfolio that replicates the systematic exposure of the optimal factor based allocation (w_f^*); B^m is a matrix of k-factor betas for the m-subset of investable assets; W is a weighting matrix with $\sum_{i=1}^{k} d_{ii} = 1$ and $0 \leq d_{ii} \leq 1$.

E. Capital Preservation

In addition to the return maximization and liability hedging objectives, (Berkelaar et al., 2010) and others argue that Central Banks have capital preservation and short-term liquidity needs. In order to incorporate this third objective, I propose a protective put approach.

The protective put strategy is implemented assuming a Black and Scholes framework, such that a Central Bank would be able to replicate a put option with a strike price equal to the current level of reserves (δ). In this setting a put option can be replicated shorting δ_P units of the underlying asset, in this case the foreign exchange reserves with value S_t, given a time horizon τ. At the same time that I invest ϕ_P at the risk-free rate. The dynamic replication can be formulated analytically as follows:

$$\begin{aligned} P_t^* &= \varphi_P + S_t \Delta_P \\ &= PV(\delta)N(-d_2) - S_t N(-d_1) \end{aligned} \tag{8}$$

Where $N(\cdot)$ is the cumulative standard normal distribution; $PV(\delta)$ is equal to the present value at the risk-free rate of the minimum safety level (δ), taking an horizon τ; d_1 and d_2 are the well-known expressions of the Black-Sholes formula.[5]

Finally, the reserves at time t will be allocated between the risk-free asset and the portfolio that replicates the systematic exposure of the optimal factor based allocation, as follows:

$$PV(\delta)N(-d_2) + S_t(1 - N(-d_1)) \tag{9}$$

In order to implement the protective put strategy, two important parameters are needed. The volatility of the portfolio that replicates the systematic exposure of the optimal factor based allocation and the put option maturity. Firstly, the volatility of the reserves' portfolio is estimated as the sample standard deviation of historical returns of the proposed asset allocation. Secondly, I assume a 1 year maturity as the relevant horizon for the Central Bank.

Finally, I propose a straightforward comparison of three competitive objectives that Central Banks desire to attain: i) Yield-give up minimization; ii) Tracking error with respect to the positive returns of the liabilities portfolio; iii) Minimization of the maximum drawdown of the reserves' portfolio. Acknowledging, that the importance of each of this objectives can be also endogenous to the relevant institutional factors of each country, I propose a simple starting point based on a

[5] As is well known from the Black-Scholes fromula, the expressions d_1 and d_2 are given by:

$$d_1 = \frac{1}{\sigma\sqrt{\tau}}(ln\frac{S_t}{\delta}) + (r + \frac{1}{2}\sigma^2)\tau$$
$$d_2 = d_1 - \sigma\sqrt{\tau}$$

Here σ is the volatility of the portfolio that replicates the systematic exposure of the optimal factor based allocation.

z-score ranking similar to one used by (Chincarini, 2006) in stock rankings. The main idea is that for each of the replicating portfolios, given by the liabilities proxy and the factor model, we calculate the z-score for the specific metric, and we calculate a total score for each of the portfolios.

III. An Illustration: the case of Chile

A. Context

Chile is a small (0.38% of World GDP at PPP in 2015), open economy (60% openness index in 2015) that is mainly exposed to commodity prices (metals and mining represent 57% of the total exports in 2014[6]). The Central Bank of Chile is an autonomous entity granted by Chile's National Constitution. According to the Basic Constitutional Act of the Central Bank of Chile, its main objectives are to safeguard the stability of the currency and the normal functioning of internal and external payments. Since 1999, the foreign Exchange policy is led by a floating exchange rate, although the Central Bank maintained the right to intervene in the foreign exchange markets.

Historically, the foreign exchange reserves were mainly used by the Chilean Central Bank to maintain the external value of the currency at a fixed rate. Nevertheless, currently the role of reserves are more related to self-insurance and foreign liquidity, (De Gregorio, 2011b).

Consistently with IMF's guidelines for foreign exchange reserve management, the Chilean Central Bank manages reserves with the following strategy: i) A liquidity tranche (24% of the total reserves); ii) A medium-term tranche (61% of the total reserves); iii) A diversification tranche (25%). These three sub-portfolios, plus cash maintained by the Chilean Treasury in the Central Bank, and other assets (special drawing rights and gold) compose the total foreign exchange reserves. In term of currency composition, the main exposures are: US Dollar (65%), Euro (17%), Australian Dollar (5%), and Canadian Dollar (5%). The average duration is 25 months.

In the following subsections we summarize historical events that are relevant for the foreign exchange reserve strategy in Chile: the 1982 Chilean Banking Crisis, the Asian Crisis, the interventions post-floating exchange rate regime, and the 2008 Financial Crisis.

1982 Financial Crisis

(Harberger, 1985) documents that foreign exchange reserves were roughly US$ 1 billion in 1978. At the end of 1979 they have grown up to US$ 2.3 billion, reaching US$ 4 billion during the period 1980-1981. In mid-1981 when Chile entered its worst economic crisis since the 1930s, the peg was at the fixed rate of 1 USD = 39 CLP. In June 1982, a 18% devaluation was announced and a further

[6]The Atlas of Economic Complexity (2014).

monthly devaluation of 0.8% was pre-announced. As of May 1983 the foreign exchange reserves had fallen -44% in YoY basis. Given the significant exposure to foreign exchange currency of the private sector, the currency depreciation lead to a collapse of the financial system. The government acted trough the Central Bank and other agencies, managing the liquidation process of financial institutions, purchasing non-performing loans, creating a program of subsidized foreign currency for debtors, mediating debt restructuring, and finally creating a recapitalization program called 'Popular Capitalism' that offered loans to acquire stocks in industrial and financial institutions. (Restrepo et al., 2009) estimate that the total cost of the 1982 Financial Crisis for the Chilean Central Bank was roughly 40% of the GDP.

Asian Crisis

(Cowan and De Gregorio, 2007) document the Chilean experience before and after the Asian Crisis. During the 1990s, the exchange rate was pegged to a band. During a period of significant capital inflows − given by the good economic expectations and the high interest rate gap between local and international yields − foreign exchange reserves grew consistently (+38% between Nov-97/Jan-96). In 1997, when the Tom Yam Kung crisis started in Thailand, a significant outflow from emerging markets occurred. The reaction of the Chilean Central bank was defending the Chilean Peso, lifting the monetary policy rate from 9% to 19%, and selling US$ 4 billion in reserves. In September 1999, after liquidating US$ 4 billion (roughly 25%) the Central Bank moved to a freely floating exchange rate.

Exchange Rate Interventions post-floating regime

Since the adoption of the fully floating regime in 1999, the Central Bank has intervened the exchange market in four occasions. As (Claro and Soto, 2013) document, in 2001 and 2002 interventions were mainly explained by the financial turmoil in Argentina and the political election in Brazil. In 2001, the intervention program consisted of spot sales of US dollars, and the program was implemented through the issuance of dollar-denominated debt. Later, in 2008 after a significant appreciation of the Chilean peso, and considering that reserves indicators (reserves to imports; reserves to M2; reserve to GDP) were relatively low, (De Gregorio, 2011a). The Central Bank decided to increase the amount of reserves in 5 percentage points of the GDP (US$ 8 billion). In September 2008, the Central Bank abandoned the plan following Lehman Brothers' collapse, after accumulating only US$ 5.75 billion (around 75% of the original reserve acquisition plan). Finally, the last intervention occurred in January 2011, the Central Bank decided to intervene, increasing its foreign exchange reserves from 13 to 17% of the GDP. A consequence of these two sterilized interventions the currency mismatch between assets (reserves in hard currencies) and liabilities (currently issued only local currency) has increased.

In September 2008, motivated by the foreign currency liquidity situation experienced after the Lehman Brothers collapse. The Chilean Treasury decided to bid US$ 700 million in US dollar deposits at 3.39%, and also injected US$ 1.05 billion in the local financial system from the Sovereign Wealth Funds. In addition, the Central Bank decided to offer liquidity in local and foreign currency though swaps and repos with local banks. As (García Cicco and Kawamura, 2014) document, the collateral assets accepted by the Central Bank, in a first phase (starting in October, 2008) were only banks deposits, while in a second phase (from January, 2009) the list was further expanded, including government bonds.

In conclusion, the Central Bank holds reserves to ensure the stability of the value of the national currency, and provide liquidity in foreign currency in specific exceptional circumstances, (of Chile, 2012). In absolute terms, Chile currently has US$ 39.7 billion, a 16% of the GDP and 27.6% of the M2. In Figure 1 we can see the three most common benchmarks used to evaluate the size of foreign exchange reserves. Chile has a level of reserves above the different rules, three months of imports is equal to 7.01% of GDP, the Guidotti rule implies a 5.66% of GDP and IMF (2011) would be equivalent to 7.4% of GDP[7].

B. Systematic Factors

As a starting point, we describe a set of factor models that mimic different global systematic risks that explain the variation of investable assets, as well as, the relevant liabilities for the Central Bank. The global version of the three factor model, (Fama and French, 2012), is a natural benchmark used in the literature. The Fama-French 3 factor model (FF3) includes: the market risk premium (MRP), Small minus Big (SMB), and High minus Low (HML). In the interpretation of (Vassalou, 2000), the market risk premium would be mainly related to surprises in GDP growth, while the SMB and the HML factors are mainly related to systematic default risk. The global version of the five factor model, (Fama and French, 2017), adds a robust versus weak profitability (RMW), and a low versus high investment (CMA) factor.[8] In a production asset pricing model, (Hou et al., 2014) show how these two new factors are a consequence of firms rational investment policies. (Chen et al., 1986) propose a macroeconomic founded factor model that takes the market risk premium (MRP), inflation surprises, industrial production surprises, the term premium (TP) and the expected inflation.[9]

[7]The IMF (2011) rule is implemented as the sum of 40% of the foreign short term debt, 5% of M3 and 5% of total exports.

[8]The global version of the Fama-French factors are available in Kenneth French's website.

[9]In our implementation of the model we measure inflation surprises and industrial production surprises using the global version of Citi's inflation and economic surprise indexes that are available in Bloomberg. The term premium (TP) is measured by the excess of high duration bonds (BofA Merrill Lynch 10+ Year US Treasury Agency Index) over short term Treasury bills (BofA Merrill Lynch US 3-Month Treasury Bill Index). The expected inflation is measured by the difference in the average yield of a nominal global

Finally, I propose a set of other factors that can be potentially important. A Economic factor, that is constructed as the excess of return of equities (MSCI ACWI Index), commodities (Bloomberg Commodity) and real estate (SP Global REIT Index) over short term Treasury bills; a Credit factor that is constructed as the excess of return of high yield bonds (BofA Merrill Lynch US High Yield Index) over investment grade corporate bonds (BofA Merrill Lynch 1-10 Year AAA-A US Corporate Government Index); a EM equities factor that is constructed as the excess of return of emerging market stocks (MSCI Emerging Markets Index) over the stocks of developed markets (MSCI World Index); a liquidity factor that is constructed from the differential return of global small (MSCI World Small Cap Index) and large stocks (MSCI World Large Cap Index); a Real Rates factor that is measured by the return of inflation linked bonds (Barclays World Inflation Linked Bonds Index); an Inflation factor that is constructed by the differential return between nominal ((BofA Merrill Lynch-Global Government Index) and inflation-linked bonds (Barclays World Inflation Linked Bonds Index); a Carry Trade factor, (Burnside et al., 2011), that is measured by the return of investing in currencies of countries with high interest rates versus low interest rate currencies (Deutsche Bank Currency Carry USD Excess Return Index); a Commodity factor that is constructed as the excess of return of commodities (Bloomberg Commodity Index) over short term Treasury bills; a Emerging Market Currency factor that is constructed as the return of a basket of relevant emerging marekt currencies against the US dollar.[10]

C. Liabilities

From the Chilean Central Bank perspective I assume that the relevant liabilities are the explicit debt outstanding, as well as, contingent liabilities related to financial sector solvency and foreign liquidity provision. The risk exposure to explicit liabilities is measured from a total return index of bonds issued by the Central Government and the Central Bank in local currency provided by Riskamerica, a private company that provides fair value pricing for the Chilean fixed income market, Figure 2.

On the other hand, I estimate the risk exposure to contingent liabilities applying the methodologies described above. Firstly, I estimate the value of the liability associated to financial sector solvency following the option pricing approach. As we have described above, relevant parameters that are needed are: i) The volatility of equity of banks, that is obtained from a GARCH (1,1) estimation on monthly returns of the MSCI Chile Banks index (1989-2016); ii) The assets and liabilities of banks, obtained annually from the Superintendency of Banks

bond index (BofA Merrill Lynch Global Government) and the inflation-linked global bond index (BofA Merrill Lynch Global Inflation-Linked Government).

[10] The emerging market currency basket is calculated as an equally weighted return between CNY/USD, BRL/USD, RUB/USD, INR/USD and ZAR/USD, such that a positive return is equal to an appreciation of EM currencies.

and Financial Institutions. In the Figure 3 I present our estimated cost of the liability as a percentage of $1 of the underlying asset. The variations in the market value of the liability related to the provision of a bailout, can be calculated as the monthly percentage of estimated value. Alternatively, we can estimate the returns related to the contingent liability of providing a financial bailout, from the absolute value of the residuals that are below the 5th percentile of residuals, obtained from the regression of the MSCI Chile Banks index on the MSCI Chile index. As we can see, this will imply that the estimated returns will follow a jump style process. In Figure 3, we document: (a) the estimated market value of the contingent liability, following the option pricing methodology; (b) the returns of the contingent liability estimated as the changes in the estimated market value; (c) the returns of the contingent liability estimated from the idiosyncratic returns methodology.

Secondly, I estimate the value of the liability associated to liquidity in foreign currency. As we have described above, relevant parameters that are needed are: i) The long-run interest rate spread in foreign currency, that is obtained from a Hodrick-Prescott filter estimation on the spread between the borrowing rate in foreign currency paid by the Chilean financial sector over the Libor-3m (1992-2016); ii) The volatility of the interest rate spread that is obtained as the sample standard deviation of the spread; iii) The external short term debt of Chile. In Figure 4 I present our estimated cost of the liability as a percentage of the GDP. The variations in the market value of the liability related to the provision of foreign, can be calculated as the monthly percentage of estimated value. Alternatively, we can estimate the returns related to the contingent liability of providing foreign liquidity from the absolute value of the returns that are below the 5th percentile of returns of the JPM EMBI Global Chile − Total Return Index. In Figure 4 panel (c), the estimation based on the returns' methodology is documented.

Finally, we construct four liabilities portfolios based on the different methodologies used to estimate each component of the contingent liabilities, and also the relative size of the different components that are assumed to construct the portfolios. Firstly, we can estimate the relative size of each components, as: i) The explicit Central Bank debt (7% of GDP); ii) The contingent liability associated to a financial bailout (0.5% of GDP), calculated as the average cost presented in Figure 3 multiplied by an average Credit to GDP (IMF) of 62%; iii) The contingent liability associated to foreign liquidity (7% of GDP), calculated as the average external debt to GDP (IMF). Alternatively, we consider a 10% weight for the explicit debt, and equal weights for the two contingent liabilities analyzed (45%).

D. Factor Models

We start from a large set of asset classes that include government and corporate fixed income, investment grade and high yield bonds, developed and emerging market bonds and equities, and derivatives. In Table 1 I estimate time series regressions for the Global Fama-French 3 Factor Model. As we can see, from these regressions we can learn from the exposure of different assets to the relevant systematic risks. For example, we can see that the MSCI Chile has a positive exposure to the three factors, and the three factors can explain 26.7% the variance of returns. While the Put Option SP index has a negative exposure to the Market Risk Premium, and positive exposure to the other two factors, and the fraction of the variance that is explained by the model is 34%.

Similarly, in Table 2 we document the time series regressions for the Global Fama-French 5 Factors. In Table 3 the Chen, Roll, and Ross Model regressions are documented

Finally, in Table 4 to Table 7 we calculate the same time series regressions for the macro-factors that are statistically related to the estimated variation of the market value of Central Banksliabilities.[11]

As we can see, certain models are either better explaining the variation of equities or fixed income assets. Therefore, having different factor models will allow us to learn how important is being able to explain the variation of traded assets versus the non-tradable liabilities.

E. Optimal Factor Allocation

Given the factor models, and the returns of the liabilities portfolios described above. In Figure 5 to 11 we document the optimal factor allocation for each factor-liabilities combination. As we can see, from the analytic solution of the optimal portfolio, the allocation depends on the expected risk premiums, the correlation among factors, and the correlation of each factor with the liabilities portfolio. For example, in Figure 5 we show that at low levels of risk aversion, significantly high long positions in the HML factor are calculated. Conversely, in Figure 6 we show that the optimal factor allocation would be a long position in the RMW factor and a short position in the HML factor. As is well known from the mean-variance optimization literature, changes in parameters have large impacts in the optimal allocation. Consequently, the decision with respect to the relevant factor model, and the risk aversion parameter, are key steps of the asset allocation process.

F. Portfolio Replication

The optimal factor allocation could not be be investable directly.However, currently there are ETFs that intend to replicate the most common risk factors such as iShares Russell Microcap Index IWC (Size), Guggenheim SP 500 Pure Value RPV (Value) or QuantShares U.S. Market Neutral Momentum Fund (Momentum). Thus, I apply the replication method proposed in Section II.D, based on

[11]The macro-models used in Table 4 to Table 7 are the combinations of at most three factors that better explain the variation ($adjusted - R^2$) of each of the proposed liabilities portfolio returns.

the asset classes' exposure to the systematic risks that are estimated by each factor model, presented in Figure 12 to Figure 17, including equities and short sales constraints. As we can see, in Figure 12 I present the asset allocation for the Fama-French 3 Factor Model by each of the proposed proxies of Central Bank's liabilities portfolio, considering only fixed income assets. As we can see, at lower level of risk aversion the optimal reserves portfolio would be 100% in High Yield Bonds, while at higher level of risk aversions the reserves' portfolio is more diversified toward government bonds of developed markets and Chinese money market instruments. Similar results are presented in Figure 13 for the Fama-French 5 Factor Model. In Figure 14, we show the fixed income replicating portfolio for the Macro Model I. As we can see, the portfolios are more exposed to US Treasuries, and the diversification is shifted to Korean money market instruments, High Yield bonds and emerging market bonds in hard currency. In Figure 15, the replicating portfolio for the Macro Model II is mainly exposed to Chinese money market instruments. In the case of the Macro Model III, Figure 16, the largest weight in the portfolios are emerging market bonds in local currency. In Figure 17, the allocation for the Macro Model IV is mainly concentrated in global inflation-linked bonds and Australian government bonds. Alternatively, we can replicate the optimal factor allocation for a broader set of asset classes. In Figure 18, I present the results for the Fama-French 3 Factor Model. As we can see, the most important asset classes are Copper, developed countries equities and put options on the SP 500. In Figure 19, I present the results for the Fama-French 5 Factor Model, the most important asset class is Japanese equities. For the Macro Model I, in Figure 20, the most important assets are swaptions and Global Government bonds. In Figure 21, the results for the Macro Model II show that the most important assets are emerging market equities and swaptions. The results presented in Figure 22 are the replicating portfolios for the Macro Model III, the most important assets are swaptions and put options on the SP 500. In Figure 23, the Macro Model IV replicating portfolios are mainly exposed to the global inflation-linked bonds.

G. Evaluation the Replicating Portfolios

As we have seen, the decision with respect to the relevant factor model, the risk measurement of the liabilities portfolio, or the risk aversion of the Central Banker influence the asset allocation significantly. In order to compare the different replicating portfolios obtained by factor model and liabilities portfolio proxy, I apply the protective put strategy proposed in the methodological section to the portfolios that are consistent with an arbitrary risk aversion of 20. Therefore, for each of the selected portfolios I estimate the in-sample volatility of the historical monthly returns. The main input to estimate the allocation in the risk-free asset that would be needed to replicate a protective put of an at-the-money put option on the risky portfolio. As we can see, in Figure 24 we document the allocation in the risk-free asset that is obtained by the proposed protective put strategy. The fixed income strategies tend to be less volatile, and consequently they require a lower investment in the risk-free asset.

Considering the different replicating portfolios weighted by their required allocation in the risk-free asset, I provide a quantitative comparison for the models that we have proposed, as well, different proxies of the liabilities that are relevant for the Central of Chile. In Figure 25, we show the relevant metrics for the fixed income spectrum, yield give-up, tracking error of liabilities, and maximum drawdown. Similarly, in Figure 25 we document the same metrics for the replicating portfolios constructed for the broader spectrum of assets.

Given the metrics, in Figure 27 we can calculate the z-score for each metric, and calculate the total score by factor model and proposed liabilities portfolio proxy (lower is better). Interestingly, we can see that the total scores tend to favor factor models that include macroeconomic factors, that are usually not supported by the data in the empirical asset pricing literature. For example, if we consider only fixed income strategies, and the Liabilities 1 and 4, the replicating portfolio with the best score is the Macro Model II; in the case of Liabilities 2 the best replicating portfolio is the Macro Model IV; for Liabilities 3 is Macro Model II. On the other hand, if we consider the entire spectrum of investable asset classes, we can see that the best replicating portfolios are the Macro Model IV, for Liabilities 1, 2 and 4, the Chen, Roll and Ross for Liabilities 3. In Figure 28, we document the composition of the risky portion of the reserves portfolio that would be selected given by the z-score rule, and a fixed income spectrum. In this case, Asia Pacific government bonds are include in the allocation, as well as, European bonds or US Treasuries in the case of Liabilities 3. Conversely, in Figure 29, we show the portfolios that would be selected if there is no restriction to the fixed income spectrum. Interestingly, the portfolio allocation change significantly. A global diversified index of inflation-linked bonds is important in three of the cases, emerging market bonds and equities, commodities such as gold, and derivatives have important weights in the allocation.

Independently of the rationality behind the amount of reserves that a Central Bank hold, countries desire to minimize the so-called 'yield give up'. In other words, they prefer to reduce the social cost of holding reserves, that tend to exist because reserves are invested in lower-yielding assets with respect to its cost of funding.

IV. Conclusion

Central Bank reserves have continued to be important in a world that is dominated by floating exchange rate systems. In this paper, we think about reserves as precautionary savings that allow Central Banks fulfill its role of lender of last resort. Something that was specially clear during the Great Recession, when they had to provide liquidity in foreign or local currency, and deal with solvency problem of financial institutions. This buffer that Central Banks hold is invested taking into account the expected return of the investments, minimizing the social cost of holding reserves, and the value of the assets at moments of financial stress. At the same time that Central Bank decisions are affected by capital preservation requirements, reputational risk, or restricted investment universe. In this paper I propose an asset allocation methodology for Central Bank reserves. I assume that Central Banks are risk averse mean-variance agents that care about a specific number of observable risk factors. As a result, we start from an institution that decide a factor allocation in an asset-liability framework. The relevant liabilities are composed by explicit debt that pays a variable cost that is observable, and two unobserved contingent liabilities that are related with a potential bailout of the banking sector and the provision of foreign liquidity. In addition, I include a capital preservation motives applying a protective put strategy. Finally, given the relevant investment universe, we search for the investable portfolio that has the desired systematic risk exposure, and balance the different objectives of the Central Bank (yield give-up minization, hedging of liabilities, and capital preservation). I illustrate the model for the case of Chile, a small open economy that is exposed to commodities. Based on different assumptions, I propose proxy-measures for the liabilities that the Central Bank of Chile faces. On the other hand, I estimate different factor models that incorporate numerous systematic risks. Importantly, we find that factor models that are based on macroeconomic factors lead to a better balance of the different objectives. I also find that, if we focus on fixed income strategies the portfolios that result include Asia Pacific government bonds, European bonds and US Treasuries. However, if a more ample spectrum of assets is considered, a more global diversified index of inflation-linked bonds is chosen, emerging market bonds and equities, commodities such as gold, and derivatives, such as swaptions or put option are also included.[12]

[12]The returns of the put option and swaptions contracts are measured from the CBOE SPX Volatility Index and the Merrill Lynch Swaption Option Volatility Estimate 3 Month Index respectively, following the methodology described in the Appendix.

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V. Appendix

A. Put Option and Swaption Returns

The put option returns are estimated from the CBOE SPX Volatility Index. Taking the historical implied volatility (σ_t) and the 3-month Treasury bill yield (r_t), the price of a 1-month at-the-money put option is calculated using the Black-Scholes formula, as follows:

$$
\begin{aligned}
P_t &= N(-d_2)exp(-\frac{r_t}{12}) - N(-d_1) \\
d_{1,t} &= \frac{1}{\sigma_t\sqrt{1/12}}(\log 1 + \frac{r_t + \sigma_t^2/2}{12}) \\
d_{2,t} &= d_1 - \sigma_t\sqrt{1/12}
\end{aligned}
\tag{10}
$$

where $N(\cdot)$ is the standard normal cumulative distribution function.

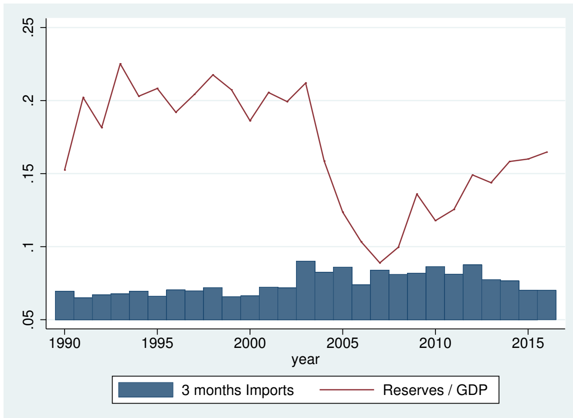
The historical returns of put options would be given by:

$$
r_{P,t} = P_{t+1}/P_t - 1 \tag{11}
$$

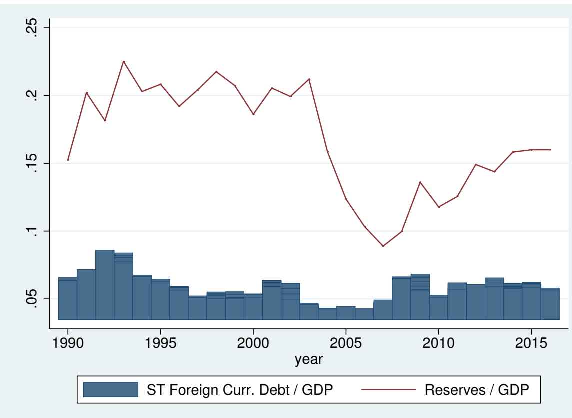
The swaption returns are estimated from the Merrill Lynch Swaption Option Volatility Estimate 3 Month Index. Taking the historical implied swaption volatility ($\sigma_{S,t}$), the 2Y/3m interest rate swap ($r_{k,t}$), and the 6m, 1Y, 1.5Y, and 2Y zero-coupon rates (r_{0,T_i}), the price of a 3-month at-the-money swaption on the 2Y/3m swap is calculated using the Black formula, as follows:

$$
\begin{aligned}
S_t &= \sum_{i=1}^{4} Z(0,T_i)(r_{k,t}N(d_{1,t}) - r_k N(d_{2,t})) \\
d_{1,t} &= \frac{1}{\sigma_{S,t}\sqrt{3/12}} \log 1 + \frac{1}{2}\sigma_{S,t}\sqrt{3/12} \\
d_{2,t} &= d_{1,t} - \sigma_{S,t}\sqrt{3/12}
\end{aligned}
\tag{12}
$$

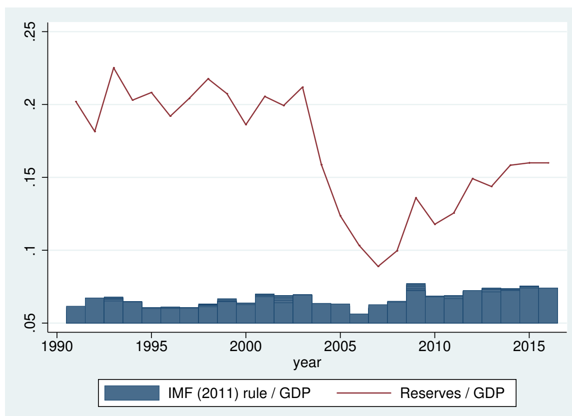
where $Z(0,T_i)$ is the discount factor at $r_{0_{Ti}}$ (4 coupon payments at time T_i). The historical returns can be calculated as the relative change in prices.



(a) Central Bank Reserves - 3 months imports rule



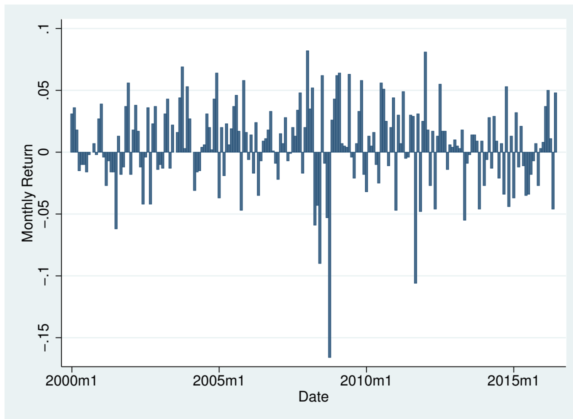
(b) Central Bank Reserves - Guidotti's rule

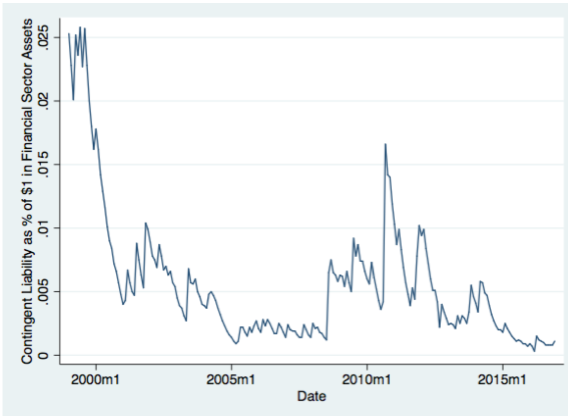


(c) Central Bank Reserves - IMF (2011)

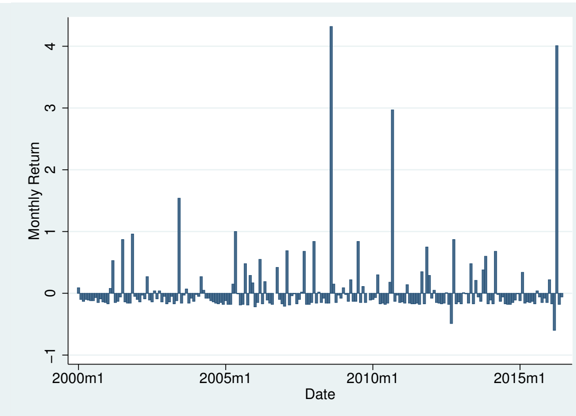
Figure 1. : Chile Central Bank Reserves - Benchmarks as a % of GDP

Figure 2. : Returns of Market Value of Debt

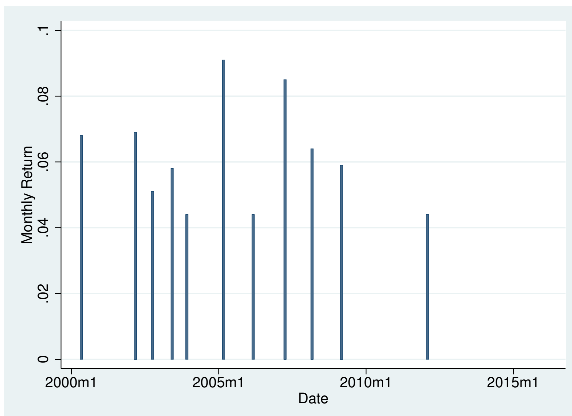




(a) Contingent Liability - Financial Bailout

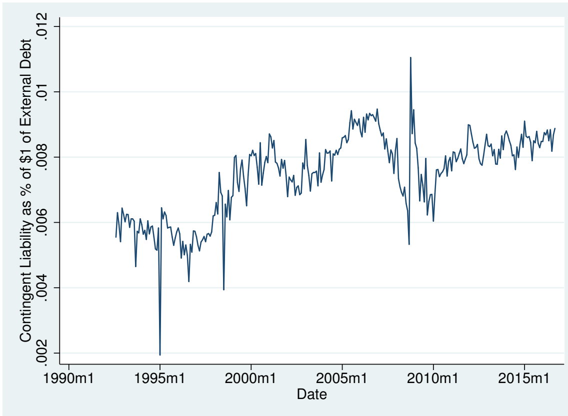


(b) Return of Contingent Liability - Financial Sector Solvency (Option Pricing Methodology)

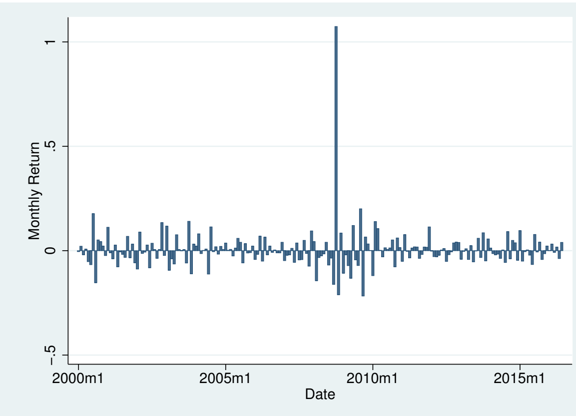


(c) Return of Contingent Liability - Financial Sector Solvency (Idiosyncratic Return Methodology)

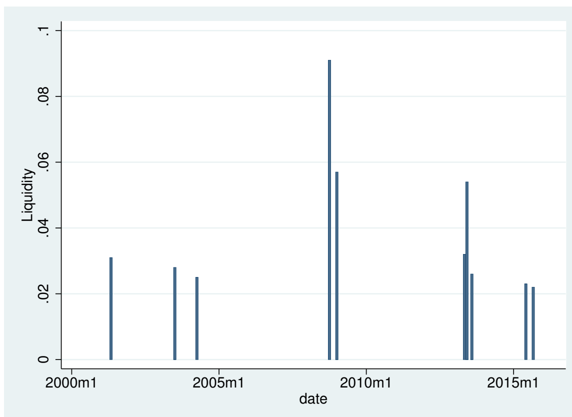
Figure 3. : Contingent Liability - Financial Sector Solvency



(a) Contingent Liability - Foreign Liquidity



(b) Return of Contingent Liability - Foreign Liquidity (Swaption Methodology)



(c) Return of Contingent Liability - Foreign Liquidity (Return Methodology)

Figure 4. : Contingent Liability - Foreign Currency Liquidity

	MSCI Chile	Bofa US Treasuries	Bofa Euro Treasuries	Bofa Japan Gov.	Bofa Swiz Gov.	Bofa UK Gov.	Bofa NZ Gov.	China Money Market	Korea Money Market	Bofa Global HY	Bofa Global Corp.
MRP	0.816***	-0.0487*	-0.0152	-0.0170	-0.0470*	-0.0474*	-0.0319*	-0.0274	0.368***	0.465***	0.194***
	(0.100)	(0.0195)	(0.0145)	(0.0129)	(0.0195)	(0.0237)	(0.0147)	(0.0458)	(0.0460)	(0.0503)	(0.0340)
SMB	0.452**	-0.0880*	-0.0667*	-0.0625	-0.0414	-0.114**	-0.0779**	-0.0477	0.246*	0.265***	0.0868
	(0.160)	(0.0356)	(0.0259)	(0.0328)	(0.0286)	(0.0392)	(0.0284)	(0.0289)	(0.121)	(0.0714)	(0.0449)
HML	0.138	-0.0139	0.0126	-0.00979	-0.0199	-0.0829	-0.0160	-0.0771	0.119	0.137*	0.122***
	(0.151)	(0.0336)	(0.0236)	(0.0260)	(0.0249)	(0.0428)	(0.0249)	(0.0752)	(0.0836)	(0.0625)	(0.0346)
Const.	0.00476	0.00535***	0.00571***	0.00350***	0.00443***	0.00737***	0.00685***	0.00331***	0.00291	0.00300*	0.00342**
	(0.00355)	(0.000752)	(0.000632)	(0.000570)	(0.000638)	(0.000970)	(0.000669)	(0.000789)	(0.00189)	(0.00139)	(0.00107)
N	313	313	313	313	313	313	313	312	312	223	235
adj. R-sq	0.266	0.036	0.012	0.012	0.032	0.030	0.023	0.001	0.195	0.574	0.247

	JPM GBI-EM	JPM EMBI	Barclays World Inflation	MSCI World Small Caps	MSCI World Large Caps	MSCI World	MSCI ACWI	MSCI EM	MSCI Asia Pacific	Bofa Canada Gov.	Bofa Australia Gov.
MRP	0.0529	0.448***	0.177***	1.063***	0.978***	0.976***	0.995***	1.196***	1.089***	-0.00874	-0.0447**
	(0.0293)	(0.0775)	(0.0473)	(0.0154)	(0.00564)	(0.00633)	(0.00793)	(0.0558)	(0.0405)	(0.0173)	(0.0171)
SMB	0.0767	0.0104	0.0928	0.792***	-0.229***	-0.191***	-0.148***	0.597***	0.181	-0.0466	-0.0690
	(0.0657)	(0.0774)	(0.0573)	(0.0421)	(0.0156)	(0.0139)	(0.0156)	(0.120)	(0.121)	(0.0359)	(0.0371)
HML	0.120*	0.0239	0.133**	0.190***	-0.0655***	0.0128	0.0138	0.0608	0.0758	-0.0520	-0.00959
	(0.0498)	(0.0756)	(0.0423)	(0.0384)	(0.0108)	(0.0112)	(0.0131)	(0.0985)	(0.0934)	(0.0348)	(0.0332)
Const.	0.00313*	0.00597**	0.00384**	0.000723	0.000149	0.000577*	0.000457	0.000922	-0.00263	0.00626***	0.00692***
	(0.00132)	(0.00217)	(0.00139)	(0.000579)	(0.000240)	(0.000246)	(0.000311)	(0.00239)	(0.00167)	(0.000774)	(0.000774)
N	210	271	235	258	266	313	313	313	313	313	313
adj. R-sq	0.023	0.274	0.143	0.964	0.992	0.990	0.985	0.637	0.720	0.002	0.022

	Bofa EMBI Plus Mexico	Nikkei	Oil	Gold	Copper	Chilean Peso	Bofa EMBI Plus Chile	Bofa Global Gov.	Put Option S&P	Swaption US 3m
MRP	0.257***	1.056***	0.394*	0.0864	0.728***	0.274***	0.334***	0.0875***	-2.886***	-0.407
	(0.0459)	(0.0624)	(0.185)	(0.0814)	(0.120)	(0.0569)	(0.0662)	(0.0248)	(0.286)	(0.282)
SMB	-0.0539	0.220	1.021***	0.477***	0.340	0.224**	0.154	-0.0277	0.210	0.656
	(0.0776)	(0.181)	(0.220)	(0.115)	(0.181)	(0.0785)	(0.0856)	(0.0528)	(0.443)	(0.464)
HML	-0.0374	-0.0341	0.168	0.00157	0.460**	0.0331	-0.0160	0.0729	0.0937	0.184
	(0.0667)	(0.143)	(0.232)	(0.0990)	(0.151)	(0.0664)	(0.0748)	(0.0460)	(0.366)	(0.359)
Const.	0.00648***	-0.00333	0.00482	0.00479	0.000171	-0.00345*	0.00178	0.00475***	0.0317**	0.00734
	(0.00179)	(0.00248)	(0.00555)	(0.00272)	(0.00395)	(0.00165)	(0.00205)	(0.00106)	(0.0105)	(0.0122)
N	271	313	313	313	313	313	229	313	313	236
adj. R-sq	0.134	0.524	0.067	0.044	0.180	0.182	0.239	0.037	0.344	0.003

Table 1—: Global Fama-French 3 Factors

Standard errors in parentheses $*p < 0.05 **p < 0.01 ***p < 0.001$

	MSCI Chile	Bofa US Treasuries	Bofa Euro Treasuries	Bofa Japan Gov.	Bofa Swiz Gov.	Bofa UK Gov.	Bofa NZ Gov.	China Money Market	Korea Money Market	Bofa Global HY	Bofa Global Corp.
MRP	0.748***	-0.00702	-0.00546	-0.0121	-0.0282	-0.00467	-0.0136	-0.0320	0.351***	0.444***	0.253***
	(0.113)	(0.0214)	(0.0173)	(0.0209)	(0.0219)	(0.0274)	(0.0160)	(0.0579)	(0.0562)	(0.0500)	(0.0370)
SMB	0.408*	-0.0348	-0.0539	-0.0548	-0.0174	-0.0544	-0.0605	-0.0408	0.265	0.282***	0.162**
	(0.165)	(0.0379)	(0.0290)	(0.0295)	(0.0326)	(0.0419)	(0.0315)	(0.0238)	(0.138)	(0.0773)	(0.0509)
HML	0.391	-0.102*	-0.000239	-0.0126	-0.0627	-0.154*	-0.0674	-0.0180	0.262	0.229**	0.0206
	(0.236)	(0.0517)	(0.0401)	(0.0444)	(0.0434)	(0.0655)	(0.0460)	(0.0525)	(0.139)	(0.0858)	(0.0709)
RMW	-0.00470	0.165*	0.0442	0.0225	0.0681	0.200**	0.0414	0.0599	0.161	0.131	0.291**
	(0.261)	(0.0646)	(0.0471)	(0.0513)	(0.0490)	(0.0698)	(0.0430)	(0.0873)	(0.129)	(0.117)	(0.0936)
CMA	-0.588	0.154*	0.0349	0.0195	0.0763	0.125	0.108	-0.102	-0.360*	-0.279*	0.0858
	(0.305)	(0.0653)	(0.0513)	(0.0582)	(0.0555)	(0.0853)	(0.0588)	(0.194)	(0.164)	(0.121)	(0.102)
Const.	0.00546	0.00447***	0.00550***	0.00340***	0.00405***	0.00640***	0.00655***	0.00324***	0.00267	0.00291	0.00196
	(0.00383)	(0.000813)	(0.000681)	(0.000565)	(0.000678)	(0.00103)	(0.000695)	(0.000905)	(0.00208)	(0.00150)	(0.00112)
N	313	313	313	313	313	313	313	312	312	223	235
adj. R-sq	0.272	0.068	0.009	0.005	0.037	0.048	0.030	-0.000	0.211	0.591	0.287

	JPM GBI-EM	JPM EMBI	Barclays World Inflation	MSCI World Small Caps	MSCI World Large Caps	MSCI World	MSCI ACWI	MSCI EM	MSCI Asia Pacific	Bofa Canada Gov.	Bofa Australia Gov.
MRP	0.116**	0.415***	0.234***	1.075***	0.969***	0.969***	0.986***	1.113***	1.159***	0.0170	-0.0264
	(0.0389)	(0.0676)	(0.0503)	(0.0172)	(0.00763)	(0.00897)	(0.0101)	(0.0598)	(0.0565)	(0.0203)	(0.0194)
SMB	0.136	-0.00230	0.180**	0.792***	-0.242***	-0.199***	-0.151***	0.571***	0.250*	-0.0163	-0.0521
	(0.0736)	(0.0868)	(0.0676)	(0.0402)	(0.0143)	(0.0118)	(0.0147)	(0.116)	(0.115)	(0.0386)	(0.0384)
HML	-0.0453	0.137	0.0602	0.00168	-0.0160	0.0753***	0.0957***	0.458**	-0.214	-0.122*	-0.0701
	(0.0858)	(0.172)	(0.0814)	(0.0537)	(0.0164)	(0.0195)	(0.0212)	(0.158)	(0.146)	(0.0521)	(0.0563)
RMW	0.183	-0.0126	0.345**	-0.132*	-0.0191	0.0188	0.0494	0.166	0.0735	0.0753	0.0263
	(0.108)	(0.152)	(0.119)	(0.0557)	(0.0224)	(0.0227)	(0.0275)	(0.176)	(0.163)	(0.0571)	(0.0522)
CMA	0.222	-0.200	0.0207	0.117	-0.0124	-0.0573*	-0.111***	-0.920***	0.449*	0.124	0.124
	(0.116)	(0.227)	(0.115)	(0.0631)	(0.0198)	(0.0267)	(0.0279)	(0.206)	(0.177)	(0.0667)	(0.0735)
Const.	0.00188	0.00634**	0.00226	0.000744	0.000370	0.000668*	0.000479	0.00133	-0.00368*	0.00578***	0.00666***
	(0.00136)	(0.00215)	(0.00153)	(0.000615)	(0.000275)	(0.000269)	(0.000339)	(0.00245)	(0.00179)	(0.000839)	(0.000794)
N	210	271	235	258	266	313	313	313	313	313	313
adj. R-sq	0.046	0.272	0.181	0.965	0.992	0.991	0.987	0.667	0.728	0.010	0.029

	Bofa EMBI Plus Mexico	Nikkei	Oil	Gold	Copper	Chilean Peso	Bofa EMBI Plus Chile	Bofa Global Gov.	Put Option S&P	Swaption US 3m
MRP	0.263***	1.177***	0.277	0.105	0.577***	0.273***	0.383***	0.152***	-2.816***	-1.062**
	(0.0485)	(0.0861)	(0.234)	(0.0937)	(0.131)	(0.0674)	(0.0839)	(0.0296)	(0.351)	(0.364)
SMB	-0.0319	0.306	0.873**	0.493***	0.206	0.211*	0.163	0.0469	0.286	-0.0426
	(0.0845)	(0.181)	(0.275)	(0.131)	(0.212)	(0.0870)	(0.0984)	(0.0568)	(0.467)	(0.533)
HML	0.00274	-0.655**	0.444	-0.165	0.964***	-0.0390	-0.214	-0.106	-0.130	1.375
	(0.124)	(0.215)	(0.397)	(0.183)	(0.229)	(0.108)	(0.117)	(0.0715)	(0.655)	(0.748)
RMW	0.124	-0.109	-0.252	-0.0629	-0.252	-0.0983	-0.0336	0.198*	0.210	-2.481*
	(0.139)	(0.240)	(0.520)	(0.208)	(0.316)	(0.144)	(0.179)	(0.0910)	(0.742)	(1.158)
CMA	-0.0802	1.062***	-0.751	0.166	-0.957**	0.0795	0.312*	0.292**	0.299	-1.756
	(0.168)	(0.266)	(0.513)	(0.237)	(0.301)	(0.135)	(0.156)	(0.0901)	(0.856)	(1.015)
Const.	0.00609**	-0.00455	0.00660	0.00459	0.00248	-0.00326	0.00128	0.00350***	0.0303**	0.0210
	(0.00191)	(0.00245)	(0.00668)	(0.00284)	(0.00419)	(0.00183)	(0.00235)	(0.00104)	(0.0114)	(0.0146)
N	271	313	313	313	313	313	229	313	313	236
adj. R-sq	0.131	0.565	0.065	0.040	0.199	0.178	0.246	0.077	0.341	0.030

Table 2—: Global Fama-French 5 Factors

	MSCI Chile	Bofa US Treasuries	Bofa Euro Treasuries	Bofa Japan Gov.	Bofa Swiz Gov.	Bofa UK Gov.	Bofa NZ Gov.	China Money Market	Korea Money Market	Bofa Global HY	Bofa Global Corp.
Inf. Surp.	-0.190	-0.0270*	0.0139	0.0124	0.00507	-0.0304	-0.0493**	-0.0650***	-0.00306	0.0283	0.00688
	(0.143)	(0.0132)	(0.0277)	(0.0162)	(0.0254)	(0.0307)	(0.0182)	(0.0184)	(0.0929)	(0.0461)	(0.0343)
Econ. Surp.	0.0145	0.00150	-0.00696	-0.00326	-0.0116	0.000785	-0.00645	-0.00310	-0.0413	-0.00399	0.00497
	(0.0495)	(0.00315)	(0.00859)	(0.00522)	(0.00813)	(0.00926)	(0.00996)	(0.00375)	(0.0336)	(0.0212)	(0.0121)
MRP	0.926***	0.00266	0.0118	-0.0101	-0.0198	0.000673	-0.0186	0.0271*	0.552***	0.549***	0.323***
	(0.137)	(0.0109)	(0.0202)	(0.0103)	(0.0247)	(0.0199)	(0.0218)	(0.0131)	(0.0785)	(0.0662)	(0.0316)
TP	-0.144	0.384***	0.236***	0.0862***	0.236***	0.445***	0.160***	-0.0103	0.138	0.0340	0.308***
	(0.201)	(0.0175)	(0.0241)	(0.0143)	(0.0332)	(0.0332)	(0.0330)	(0.0125)	(0.107)	(0.107)	(0.0404)
Expt. Inf.	-2.090	-0.0226	-0.247	0.0482	-0.0654	-0.233	-0.348	0.304**	0.417	-0.309	-0.310
	(1.287)	(0.110)	(0.185)	(0.140)	(0.243)	(0.234)	(0.206)	(0.101)	(1.100)	(0.795)	(0.273)
Const.	0.0311	0.00207	0.00589*	0.000690	0.00292	0.00604	0.00964**	0.000501	-0.00586	0.00710	0.00425
	(0.0186)	(0.00147)	(0.00238)	(0.00187)	(0.00348)	(0.00309)	(0.00293)	(0.00131)	(0.0149)	(0.0115)	(0.00382)
N	163	163	163	163	163	163	163	162	162	163	163
adj. R-sq	0.403	0.873	0.408	0.183	0.474	0.667	0.381	0.158	0.441	0.648	0.610

	JPM GBI-EM	JPM EMBI	Barclays World Inflation	MSCI World Small Caps	MSCI World Large Caps	MSCI World	MSCI ACWI	MSCI EM	MSCI Asia Pacific	Bofa Canada Gov.	Bofa Australia Gov.
Inf. Surp.	-0.0422	-0.0506	-0.0285	-0.0385	-0.0186	-0.0168	-0.0207	-0.0639	0.0204	-0.0279	-0.0475*
	(0.0460)	(0.0403)	(0.0497)	(0.0333)	(0.0129)	(0.00946)	(0.0110)	(0.0891)	(0.0654)	(0.0178)	(0.0197)
Econ. Surp.	0.0102	-0.0277	0.00498	0.00835	0.00151	0.00290	0.00235	-0.00890	0.00605	-0.00470	-0.00966
	(0.0119)	(0.0157)	(0.0161)	(0.0115)	(0.00455)	(0.00349)	(0.00404)	(0.0284)	(0.0187)	(0.00468)	(0.00773)
MRP	0.151***	0.437***	0.342***	1.123***	0.958***	0.978***	1.010***	1.305***	1.013***	0.0118	-0.0439*
	(0.0320)	(0.0554)	(0.0428)	(0.0289)	(0.0119)	(0.00815)	(0.00863)	(0.0599)	(0.0440)	(0.0141)	(0.0169)
TP	0.388***	0.359***	0.415***	-0.0376	0.00420	0.00636	0.0110	0.0586	0.105	0.296***	0.214***
	(0.0418)	(0.0823)	(0.0626)	(0.0363)	(0.0130)	(0.00916)	(0.0119)	(0.0876)	(0.0569)	(0.0192)	(0.0234)
Expt. Inf.	-0.557	-0.252	0.403	-0.532	0.311**	0.218**	0.125	-0.737	-0.639	-0.143	-0.0711
	(0.296)	(0.529)	(0.458)	(0.322)	(0.112)	(0.0807)	(0.0819)	(0.627)	(0.493)	(0.159)	(0.164)
Const.	0.00818*	0.00669	-0.00456	0.00816	-0.00528***	-0.00388***	-0.00285**	0.00897	0.00562	0.00465*	0.00571*
	(0.00397)	(0.00794)	(0.00632)	(0.00432)	(0.00150)	(0.00106)	(0.00102)	(0.00839)	(0.00682)	(0.00216)	(0.00231)
N	163	163	163	163	163	163	163	163	163	163	163
adj. R-sq	0.416	0.605	0.547	0.935	0.987	0.993	0.991	0.758	0.800	0.719	0.549

	Bofa EMBI Plus Mexico	Nikkei	Oil	Gold	Copper	Chilean Peso	Bofa EMBI Plus Chile	Bofa Global Gov.	Put Option S&P	Swaption US 3m
Inf. Surp.	-0.0733	0.0401	-0.00515	-0.169	-0.123	-0.0670	-0.0743	-0.0467	1.055*	0.669
	(0.0373)	(0.0833)	(0.224)	(0.157)	(0.192)	(0.0982)	(0.0997)	(0.0460)	(0.522)	(0.483)
Econ. Surp.	-0.0169	0.00487	0.0901	0.0372	0.118	-0.00387	-0.0109	0.00760	0.238	0.0885
	(0.0173)	(0.0285)	(0.0719)	(0.0427)	(0.0678)	(0.0286)	(0.0283)	(0.0117)	(0.140)	(0.180)
MRP	0.318***	0.844***	0.695***	0.296*	0.855***	0.422***	0.420***	0.177***	-3.860***	-1.452**
	(0.0361)	(0.0657)	(0.200)	(0.127)	(0.199)	(0.0928)	(0.0947)	(0.0315)	(0.474)	(0.488)
TP	0.516***	0.133	-0.490*	0.443*	-0.412	0.0926	0.0927	0.382***	0.0562	-3.520***
	(0.0581)	(0.0812)	(0.198)	(0.187)	(0.236)	(0.125)	(0.127)	(0.0414)	(0.395)	(0.843)
Expt. Inf.	-0.234	-0.698	3.182	-1.006	-0.719	-0.591	-0.518	-0.525	5.864	1.674
	(0.513)	(0.822)	(1.994)	(1.327)	(1.978)	(0.877)	(0.905)	(0.286)	(2.972)	(5.204)
Const.	0.00533	0.00735	-0.0361	0.0192	0.0156	0.00570	0.00809	0.00769*	-0.0413	0.0103
	(0.00681)	(0.0113)	(0.0277)	(0.0185)	(0.0280)	(0.0126)	(0.0130)	(0.00383)	(0.0400)	(0.0745)
N	163	163	163	163	163	163	163	163	163	163
adj. R-sq	0.620	0.573	0.215	0.075	0.285	0.241	0.236	0.421	0.453	0.282

Table 3—: Chen, Roll, and Ross Model

	MSCI Chile	Bofa US Treasuries	Bofa Euro Treasuries	Bofa Japan Gov.	Bofa Swiz Gov.	Bofa UK Gov.	Bofa NZ Gov.	China Money Market	Korea Money Market	Bofa Global HY	Bofa Global Corp.
Economic	0.602***	-0.0222	0.0247	-0.00539	-0.00778	0.0176	0.0135	-0.0109	0.447***	0.377***	0.174***
	(0.176)	(0.0373)	(0.0229)	(0.0199)	(0.0240)	(0.0435)	(0.0261)	(0.0143)	(0.0805)	(0.0657)	(0.0462)
Inflation	-0.0379	0.0714	0.0627	0.0615	0.0178	0.0635	0.146*	-0.00494	-0.178	-0.597***	-0.0455
	(0.372)	(0.0893)	(0.0581)	(0.0394)	(0.0974)	(0.104)	(0.0701)	(0.0248)	(0.244)	(0.157)	(0.128)
EM Curr.	1.041***	-0.0499	-0.0657**	-0.00902	-0.0724**	-0.120**	-0.0612*	0.0439*	0.295*	0.251***	0.162*
	(0.251)	(0.0310)	(0.0237)	(0.0259)	(0.0271)	(0.0446)	(0.0293)	(0.0177)	(0.115)	(0.0684)	(0.0636)
Const.	0.0101**	0.00426***	0.00431***	0.00225***	0.00310***	0.00512***	0.00544***	0.00366***	0.00440	0.00605***	0.00535***
	(0.00340)	(0.000856)	(0.000738)	(0.000486)	(0.000770)	(0.00106)	(0.000640)	(0.000359)	(0.00236)	(0.00127)	(0.00102)
N	234	234	234	234	234	234	234	234	234	222	234
adj. R-sq	0.397	0.023	0.017	0.005	0.029	0.031	0.064	0.031	0.256	0.580	0.263

	JPM GBI-EM	JPM EMBI	Barclays World Inflation	MSCI World Small Caps	MSCI World Large Caps	MSCI World	MSCI ACWI	MSCI EM	MSCI Asia Pacific	Bofa Canada Gov.	Bofa Australia Gov.
Economic	0.0321	0.339***	0.0985	0.883***	0.829***	0.840***	0.841***	0.909***	0.747***	0.0109	0.00670
	(0.0573)	(0.0910)	(0.0543)	(0.0739)	(0.0692)	(0.0669)	(0.0715)	(0.152)	(0.0985)	(0.0322)	(0.0277)
Inflation	0.476***	-0.0117	-0.590***	-0.290	-0.233	-0.246	-0.266	-0.353	-0.149	0.0693	0.229***
	(0.129)	(0.227)	(0.122)	(0.162)	(0.149)	(0.145)	(0.149)	(0.277)	(0.249)	(0.0676)	(0.0632)
EM Curr.	0.280***	0.459*	0.140*	0.502***	0.342**	0.353**	0.413***	1.030***	0.609***	-0.0569	-0.0730
	(0.0637)	(0.190)	(0.0635)	(0.116)	(0.113)	(0.108)	(0.117)	(0.240)	(0.150)	(0.0388)	(0.0422)
Const.	0.00522***	0.00993***	0.00521***	0.00791***	0.00458*	0.00489*	0.00516**	0.00924**	0.00447	0.00444***	0.00520***
	(0.00125)	(0.00149)	(0.00119)	(0.00226)	(0.00196)	(0.00191)	(0.00193)	(0.00300)	(0.00266)	(0.000705)	(0.000696)
N	209	234	234	234	234	234	234	234	234	234	234
adj. R-sq	0.149	0.382	0.300	0.603	0.604	0.626	0.642	0.586	0.499	0.016	0.116

	Bofa EMBI Plus Mexico	Nikkei	Oil	Gold	Copper	Chilean Peso	Bofa EMBI Plus Chile	Bofa Global Gov.	Put Option S&P	Swaption US 3m
Economic	0.249***	0.682***	0.0200	0.0138	0.514**	0.146	0.153	0.0985	-2.858***	-0.503
	(0.0575)	(0.0957)	(0.275)	(0.140)	(0.177)	(0.0936)	(0.0987)	(0.0543)	(0.466)	(0.456)
Inflation	0.0937	-0.0654	-1.448*	0.156	-0.518	-0.0455	-0.0744	0.410***	0.157	-0.113
	(0.133)	(0.314)	(0.572)	(0.422)	(0.474)	(0.261)	(0.287)	(0.122)	(1.077)	(1.153)
EM Curr.	0.263*	0.360*	0.847*	0.493**	0.832***	0.517***	0.471***	0.140*	-1.799**	-1.156*
	(0.104)	(0.147)	(0.416)	(0.168)	(0.215)	(0.114)	(0.119)	(0.0635)	(0.577)	(0.534)
Const.	0.00912***	0.00240	0.0105	0.00950**	0.00987*	0.00122	0.00556**	0.00521***	0.0134	-0.000379
	(0.00141)	(0.00342)	(0.00606)	(0.00342)	(0.00468)	(0.00179)	(0.00191)	(0.00119)	(0.0119)	(0.0120)
N	234	234	234	234	234	234	228	234	234	234
adj. R-sq	0.253	0.265	0.122	0.061	0.235	0.305	0.272	0.113	0.334	0.040

Table 4—: Macro Model I

	MSCI Chile	Bofa US Treasuries	Bofa Euro Treasuries	Bofa Japan Gov.	Bofa Swiz Gov.	Bofa UK Gov.	Bofa NZ Gov.	China Money Market	Korea Money Market	Bofa Global HY	Bofa Global Corp.
Economic	0.450***	0.103***	0.0601**	0.0271	0.0314	0.102*	0.0387	-0.0309	0.432***	0.108***	0.228***
	(0.125)	(0.0270)	(0.0207)	(0.0204)	(0.0235)	(0.0399)	(0.0318)	(0.0453)	(0.0796)	(0.0238)	(0.0392)
Credit	0.646***	-0.263***	-0.117***	-0.0597*	-0.113**	-0.253***	-0.101*	-0.00718	0.124	0.863***	0.0155
	(0.171)	(0.0415)	(0.0330)	(0.0247)	(0.0423)	(0.0532)	(0.0457)	(0.0159)	(0.109)	(0.0389)	(0.0690)
EM	0.699***	-0.00279	0.00507	0.00758	0.0123	0.0312	-0.0140	0.0416	0.112**	0.0319	0.0303
	(0.0778)	(0.0147)	(0.0150)	(0.0170)	(0.0138)	(0.0230)	(0.0156)	(0.0357)	(0.0419)	(0.0170)	(0.0280)
Const.	0.00567*	0.00573***	0.00590***	0.00353***	0.00444***	0.00738***	0.00694***	0.00299*	0.00412*	0.00404***	0.00441***
	(0.00288)	(0.000665)	(0.000600)	(0.000571)	(0.000620)	(0.000920)	(0.000638)	(0.00116)	(0.00182)	(0.000641)	(0.00106)
N	317	317	317	317	317	317	317	317	317	222	234
adj. R-sq	0.448	0.173	0.037	0.004	0.037	0.076	0.032	-0.001	0.221	0.895	0.217

	JPM GBI-EM	JPM EMBI	Barclays World Inflation	MSCI World Small Caps	MSCI World Large Caps	MSCI World	MSCI ACWI	MSCI EM	MSCI Asia Pacific	Bofa Canada Gov.	Bofa Australia Gov.
Economic	0.194***	0.361***	0.238***	0.691***	0.737***	0.886***	0.891***	0.886***	0.992***	0.0988***	0.0728*
	(0.0483)	(0.0544)	(0.0525)	(0.0858)	(0.0796)	(0.0681)	(0.0689)	(0.0681)	(0.107)	(0.0252)	(0.0287)
Credit	-0.256***	0.131	-0.0216	0.753***	0.542***	0.446***	0.448***	0.446***	0.258	-0.176***	-0.185***
	(0.0619)	(0.101)	(0.0857)	(0.110)	(0.101)	(0.0898)	(0.0900)	(0.0898)	(0.142)	(0.0367)	(0.0373)
EM	0.0693	0.362***	0.0338	0.173***	-0.00104	-0.0483	0.0110	0.952***	0.143	0.0139	-0.0101
	(0.0388)	(0.0717)	(0.0374)	(0.0480)	(0.0458)	(0.0412)	(0.0421)	(0.0412)	(0.0819)	(0.0181)	(0.0192)
Const.	0.00385**	0.00779***	0.00488***	0.00412*	0.00246	0.00306*	0.00291	0.00306*	0.000240	0.00635***	0.00713***
	(0.00126)	(0.00177)	(0.00134)	(0.00181)	(0.00165)	(0.00152)	(0.00154)	(0.00152)	(0.00254)	(0.000744)	(0.000711)
N	209	270	234	257	265	317	317	317	317	317	317
adj. R-sq	0.085	0.397	0.137	0.670	0.626	0.633	0.641	0.845	0.414	0.058	0.081

	Bofa EMBI Plus Mexico	Nikkei	Oil	Gold	Copper	Chilean Peso	Bofa EMBI Plus Chile	Bofa Global Gov.	Put Option S&P	Swaption US 3m
Economic	0.308***	0.976***	0.0334	0.214	0.475**	0.164*	0.152	0.249***	-2.814***	-1.664**
	(0.0508)	(0.127)	(0.217)	(0.116)	(0.149)	(0.0689)	(0.0781)	(0.0398)	(0.465)	(0.542)
Credit	-0.0884	0.197	0.487	-0.262	0.530**	0.256*	0.240*	-0.258***	-1.367*	1.669*
	(0.0636)	(0.182)	(0.346)	(0.189)	(0.202)	(0.101)	(0.119)	(0.0526)	(0.597)	(0.646)
EM	0.246***	-0.114	0.285*	0.274***	0.297**	0.166***	0.229***	-0.0125	-0.195	-0.396
	(0.0557)	(0.0974)	(0.128)	(0.0581)	(0.0953)	(0.0341)	(0.0452)	(0.0242)	(0.264)	(0.275)
Const.	0.00763***	-0.000275	0.00503	0.00470	0.00316	-0.00317*	0.00243	0.00591***	0.0235*	0.00470
	(0.00157)	(0.00331)	(0.00538)	(0.00259)	(0.00386)	(0.00146)	(0.00185)	(0.000995)	(0.0105)	(0.0116)
N	270	317	317	317	317	317	228	317	317	235
adj. R-sq	0.229	0.241	0.038	0.068	0.171	0.242	0.268	0.117	0.277	0.055

Table 5—: Macro Model II

	MSCI Chile	Bofa US Treasuries	Bofa Euro Treasuries	Bofa Japan Gov.	Bofa Swiz Gov.	Bofa UK Gov.	Bofa NZ Gov.	China Money Market	Korea Money Market	Bofa Global HY	Bofa Global Corp.
EM	0.872***	-0.0326	-0.00889	0.00582	-0.00487	0.00285	-0.0334	-0.0324	0.183***	0.111	-0.00215
	(0.0851)	(0.0177)	(0.0157)	(0.0178)	(0.0143)	(0.0242)	(0.0173)	(0.0374)	(0.0528)	(0.0579)	(0.0373)
EM Curr.	0.292	-0.0107	-0.0234	-0.00186	-0.0362	-0.0487	-0.0168	0.264	0.245**	0.486***	0.277**
	(0.244)	(0.0244)	(0.0207)	(0.0326)	(0.0199)	(0.0382)	(0.0265)	(0.219)	(0.0841)	(0.135)	(0.0843)
Const.	0.00836*	0.00437***	0.00484***	0.00273***	0.00337***	0.00538***	0.00553***	0.00576***	0.00661**	0.00816***	0.00619***
	(0.00335)	(0.000762)	(0.000669)	(0.000553)	(0.000648)	(0.000959)	(0.000619)	(0.00122)	(0.00223)	(0.00145)	(0.000965)
N	275	275	275	275	275	275	275	275	275	222	234
adj. R-sq	0.366	0.007	0.001	-0.007	0.007	0.002	0.016	0.159	0.105	0.322	0.177
	JPM GBI-EM	**JPM EMBI**	**Barclays World Inflation**	**MSCI World Small Caps**	**MSCI World Large Caps**	**MSCI World**	**MSCI ACWI**	**MSCI EM**	**MSCI Asia Pacific**	**Bofa Canada Gov.**	**Bofa Australia Gov.**
EM	-0.0441	0.351***	-0.00987	0.211**	0.0187	0.139*	0.206**	1.139***	0.431***	0.00627	-0.0316
	(0.0424)	(0.0767)	(0.0428)	(0.0811)	(0.0775)	(0.0652)	(0.0651)	(0.0652)	(0.0895)	(0.0197)	(0.0207)
EM Curr.	0.252***	0.334*	0.291**	0.916***	0.814***	0.450**	0.457**	0.450**	0.383	-0.0170	-0.0178
	(0.0579)	(0.139)	(0.0877)	(0.206)	(0.200)	(0.162)	(0.166)	(0.162)	(0.197)	(0.0347)	(0.0326)
Const.	0.00494***	0.0113***	0.00669***	0.0118***	0.00953***	0.00910***	0.00907***	0.00910***	0.00526	0.00501***	0.00537***
	(0.00127)	(0.00158)	(0.00122)	(0.00248)	(0.00219)	(0.00236)	(0.00237)	(0.00236)	(0.00277)	(0.000714)	(0.000711)
N	209	270	234	257	265	275	275	275	275	275	275
adj. R-sq	0.069	0.336	0.127	0.339	0.260	0.164	0.197	0.657	0.232	-0.006	0.009
	Bofa EMBI Plus Mexico	**Nikkei**	**Oil**	**Gold**	**Copper**	**Chilean Peso**	**Bofa EMBI Plus Chile**	**Bofa Global Gov.**	**Put Option S&P**	**Swaption US 3m**	
EM	0.209***	0.162	0.428**	0.250***	0.443***	0.185***	0.174**	-0.0172	-0.570	0.113	
	(0.0587)	(0.106)	(0.146)	(0.0688)	(0.112)	(0.0393)	(0.0563)	(0.0277)	(0.340)	(0.304)	
EM Curr.	0.240**	0.266	0.373	0.217*	0.477*	0.283***	0.448***	0.0844*	-1.945**	-1.533*	
	(0.0806)	(0.238)	(0.238)	(0.0886)	(0.198)	(0.0800)	(0.126)	(0.0398)	(0.633)	(0.611)	
Const.	0.00980***	0.00328	0.0113*	0.00736**	0.0107*	0.00134	0.00554**	0.00536***	0.00324	-0.00258	
	(0.00161)	(0.00354)	(0.00550)	(0.00280)	(0.00433)	(0.00154)	(0.00167)	(0.00111)	(0.0119)	(0.0113)	
N	270	275	275	275	275	275	228	275	275	235	
adj. R-sq	0.205	0.042	0.067	0.091	0.129	0.222	0.290	0.013	0.115	0.039	

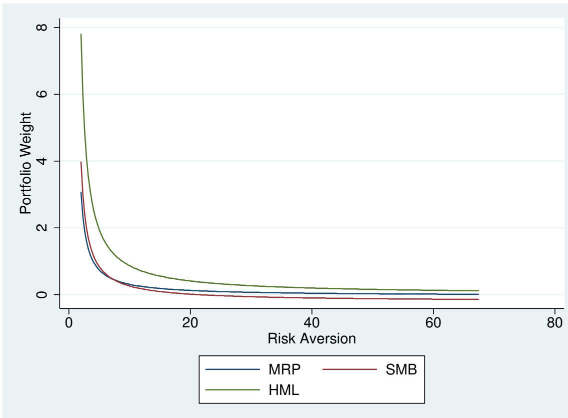
Table 6—: Macro Model III

	MSCI Chile	Bofa US Treasuries	Bofa Euro Treasuries	Bofa Japan Gov.	Bofa Swiz Gov.	Bofa UK Gov.	Bofa NZ Gov.	China Money Market	Korea Money Market	Bofa Global HY	Bofa Global Corp.
Real Rates	1.228***	0.0581*	0.0194	0.0215	-0.0171	0.000734	-0.00339	0.0413*	0.993***	0.919***	0.714***
	(0.275)	(0.0286)	(0.0358)	(0.0249)	(0.0392)	(0.0365)	(0.0360)	(0.0176)	(0.148)	(0.136)	(0.0459)
TP	-0.964***	0.394***	0.246***	0.0720***	0.248***	0.443***	0.204***	-0.0272	-0.382***	-0.429***	0.00474
	(0.161)	(0.0187)	(0.0237)	(0.0159)	(0.0270)	(0.0295)	(0.0307)	(0.0140)	(0.106)	(0.0590)	(0.0320)
Const.	0.00360	0.00221***	0.00332***	0.00177***	0.00234***	0.00354***	0.00466***	0.00334***	0.000217	0.00304	0.00102
	(0.00419)	(0.000340)	(0.000577)	(0.000504)	(0.000613)	(0.000675)	(0.000570)	(0.000351)	(0.00248)	(0.00169)	(0.000608)
N	234	234	234	234	234	234	234	234	234	222	234
adj. R-sq	0.179	0.868	0.433	0.078	0.400	0.617	0.333	0.021	0.209	0.393	0.740

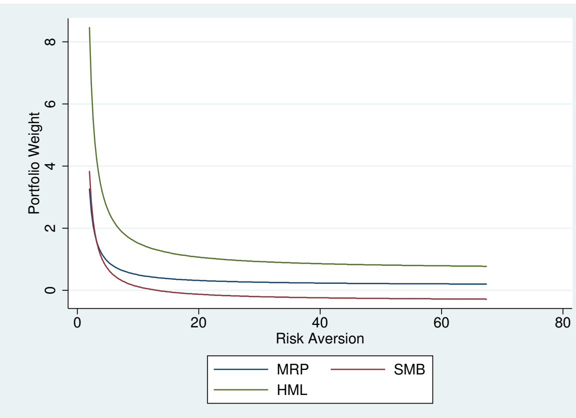
	JPM GBI-EM	JPM EMBI	Barclays World Inflation	MSCI World Small Caps	MSCI World Large Caps	MSCI World	MSCI ACWI	MSCI EM	MSCI Asia Pacific	Bofa Canada Gov.	Bofa Australia Gov.
Real Rates	0.573***	0.575***	1	1.272***	1.120***	1.140***	1.192***	1.624***	1.346***	0.0312	-0.0629
	(0.0959)	(0.147)	(.)	(0.198)	(0.155)	(0.161)	(0.167)	(0.254)	(0.168)	(0.0304)	(0.0393)
TP	0.189***	-0.0392	0	-0.864***	-0.745***	-0.754***	-0.779***	-1.047***	-0.731***	0.307***	0.282***
	(0.0323)	(0.0779)	(.)	(0.116)	(0.103)	(0.103)	(0.104)	(0.144)	(0.114)	(0.0199)	(0.0262)
Const.	0.000253	0.00504*	0	0.00409	0.00171	0.00192	0.00175	0.00175	-0.00123	0.00303***	0.00433***
	(0.00101)	(0.00204)	(.)	(0.00308)	(0.00264)	(0.00264)	(0.00268)	(0.00409)	(0.00306)	(0.000444)	(0.000592)
N	209	234	234	234	234	234	234	234	234	234	234
adj. R-sq	0.652	0.117	1.000	0.288	0.291	0.300	0.310	0.251	0.267	0.659	0.439

	Bofa EMBI Plus Mexico	Nikkei	Oil	Gold	Copper	Chilean Peso	Bofa EMBI Plus Chile	Bofa Global Gov.	Put Option S&P	Swaption US 3m
Real Rates	0.404***	1.043***	1.875***	0.966***	1.780***	0.741***	0.776***	0.600***	-2.517**	-2.923**
	(0.0853)	(0.187)	(0.345)	(0.198)	(0.351)	(0.158)	(0.163)	(0.0828)	(0.908)	(0.979)
TP	0.254***	-0.500***	-1.240***	0.0197	-1.343***	-0.324***	-0.340***	0.161***	2.242***	-2.206***
	(0.0646)	(0.137)	(0.203)	(0.170)	(0.165)	(0.0859)	(0.0890)	(0.0326)	(0.531)	(0.491)
Const.	0.00454**	-0.00168	0.00382	0.00167	0.00387	-0.00358	0.000792	0.000312	0.0220	0.0312*
	(0.00140)	(0.00366)	(0.00586)	(0.00293)	(0.00482)	(0.00201)	(0.00206)	(0.000919)	(0.0148)	(0.0129)
N	234	234	234	234	234	234	228	234	234	234
adj. R-sq	0.265	0.128	0.176	0.171	0.280	0.212	0.224	0.648	0.076	0.317

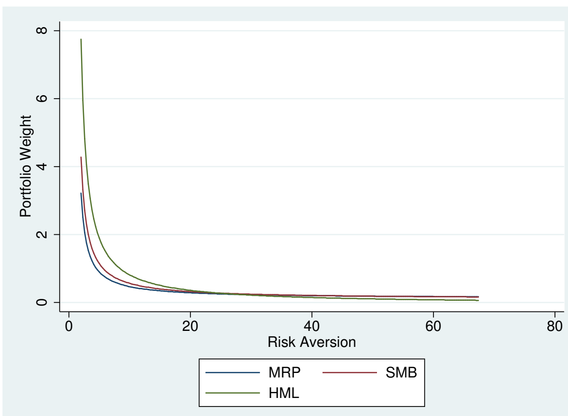
Table 7—: Macro Model IV



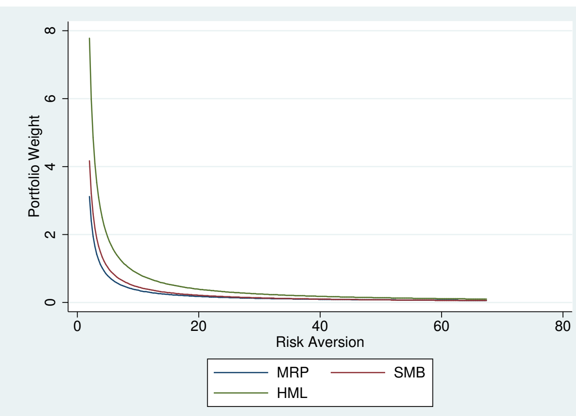
(a) Liabilities I



(b) Liabilities II

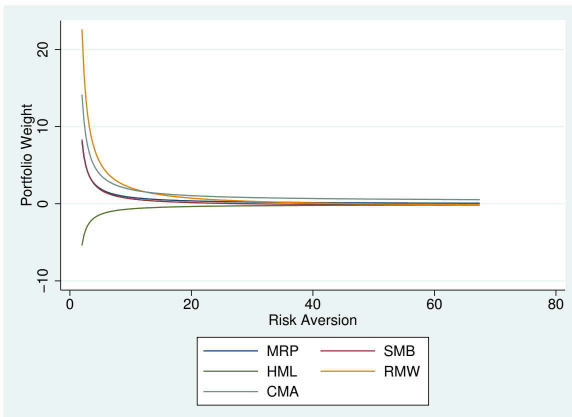


(c) Liabilities III

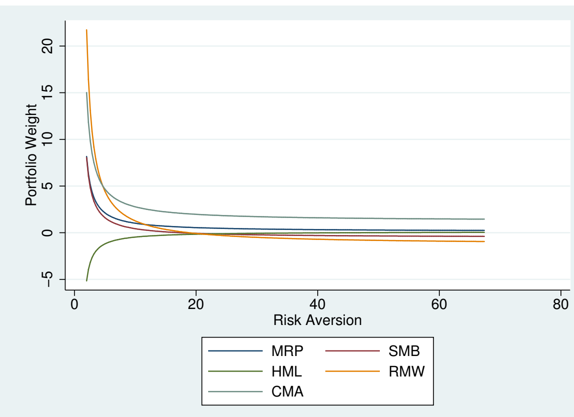


(d) Liabilities IV

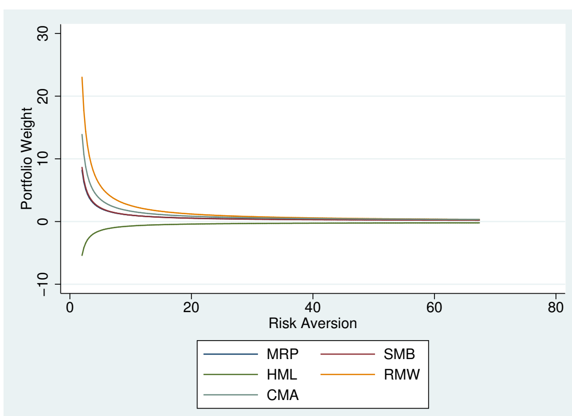
Figure 5. : Optimal Factor Allocation / Fama-French 3 Factor Model by Liabilities Portfolio



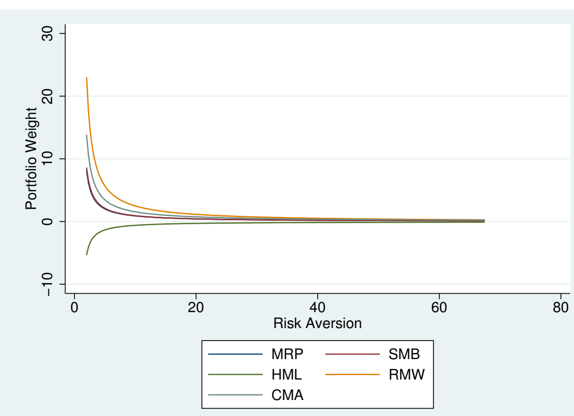
(a) Liabilities I



(b) Liabilities II

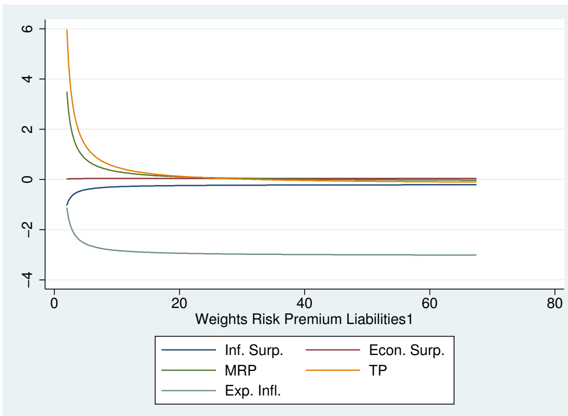


(c) Liabilities III

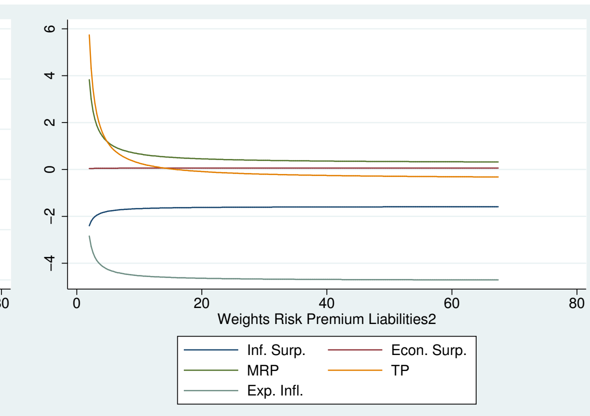


(d) Liabilities IV

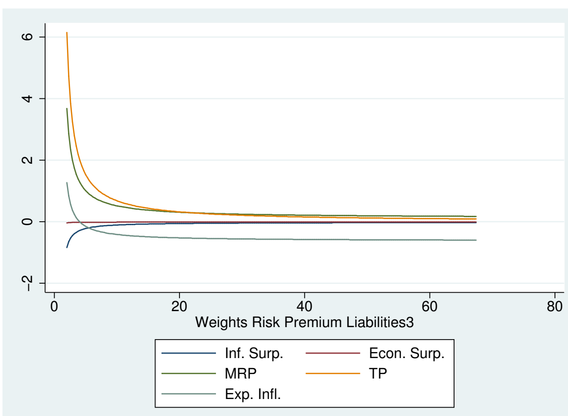
Figure 6. : Optimal Factor Allocation / Fama-French 5 Factor Model by Liabilities Portfolio



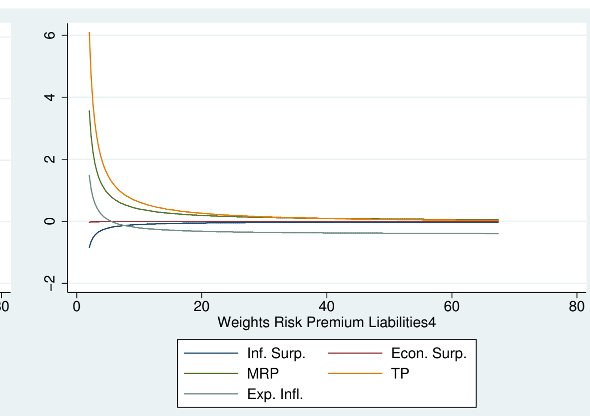
(a) Liabilities I



(b) Liabilities II

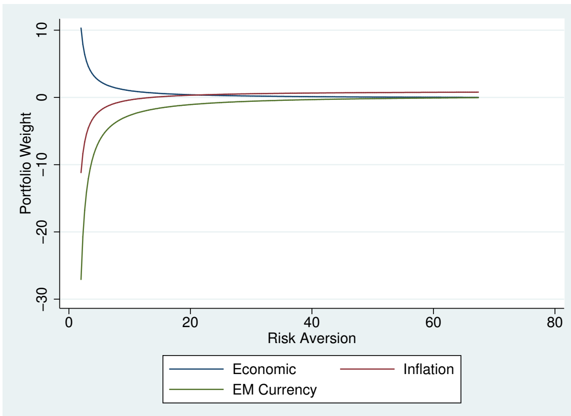


(c) Liabilities III

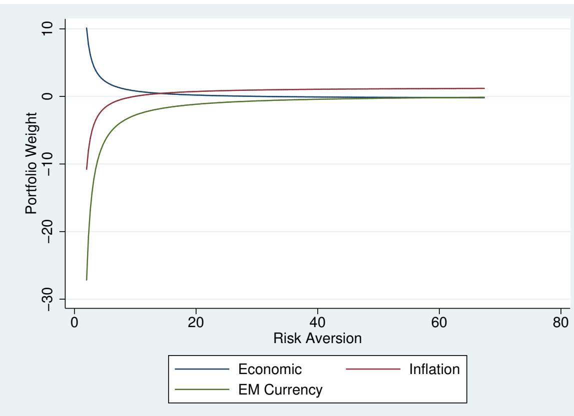


(d) Liabilities IV

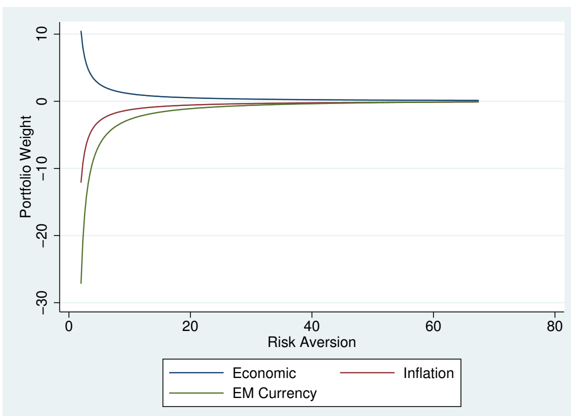
Figure 7. : Optimal Factor Allocation / Chen, Roll Ross Factor Model by Liabilities Portfolio



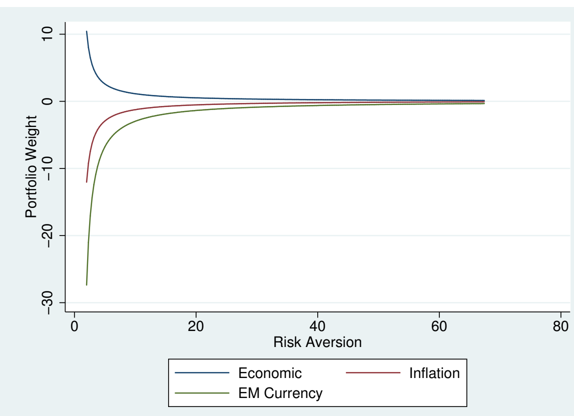
(a) Liabilities I



(b) Liabilities II

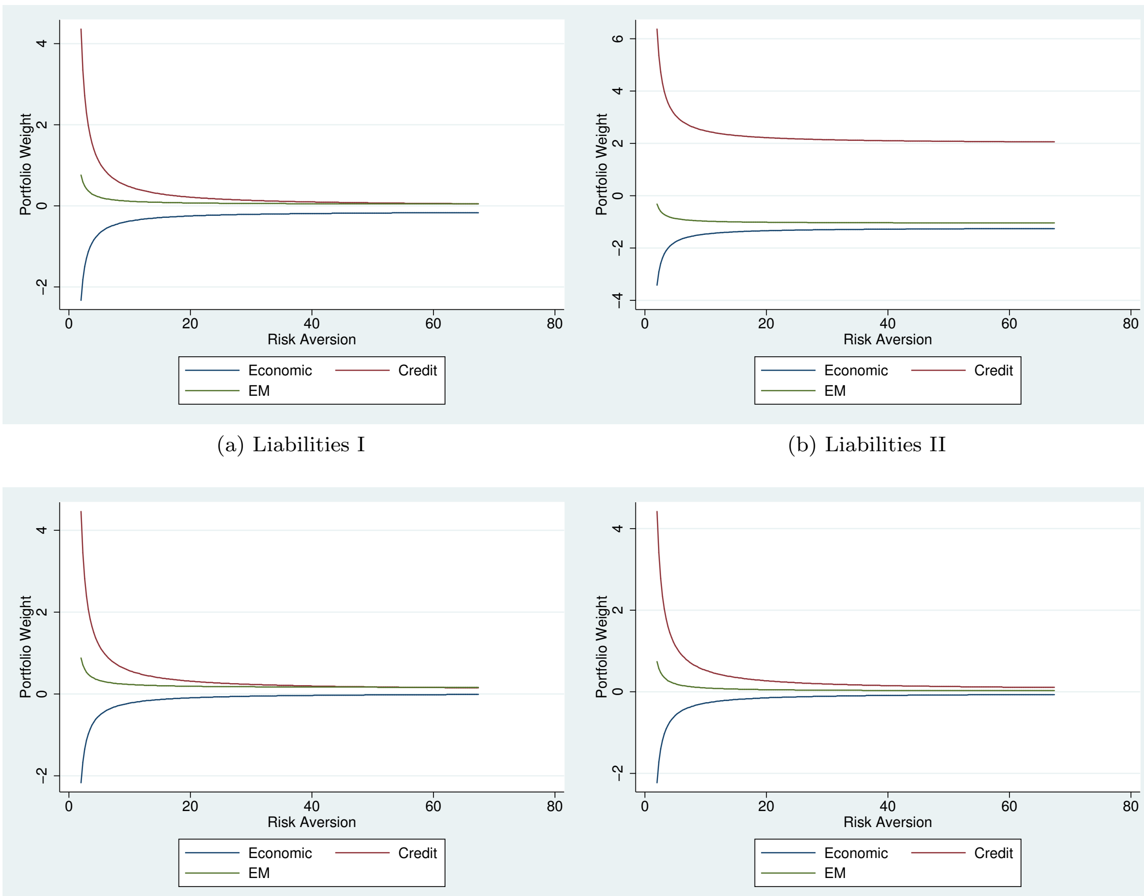


(c) Liabilities III



(d) Liabilities IV

Figure 8. : Optimal Factor Allocation / Macro Model I by Liabilities Portfolio



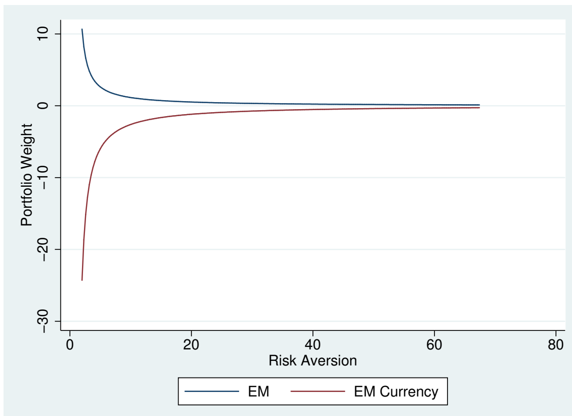
(a) Liabilities I

(b) Liabilities II

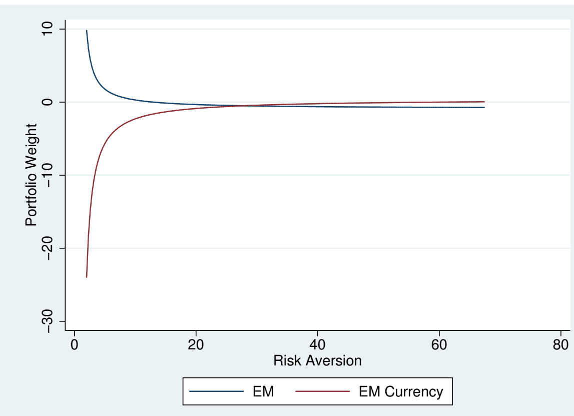
(c) Liabilities III

(d) Liabilities IV

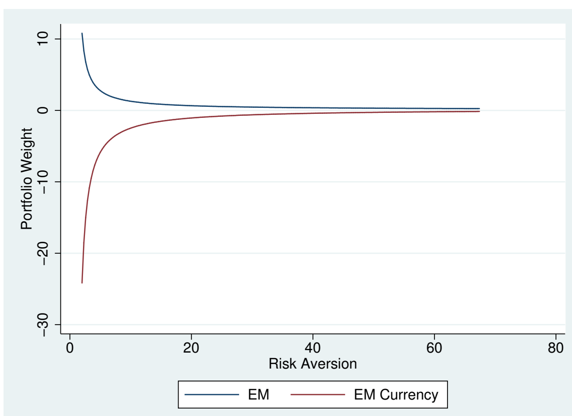
Figure 9. : Optimal Factor Allocation / Macro Model II by Liabilities Portfolio



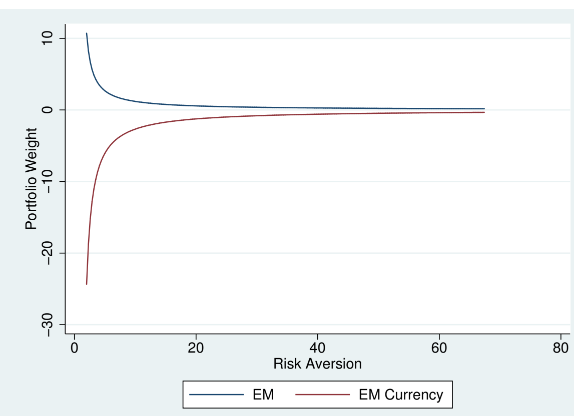
(a) Liabilities I



(b) Liabilities II

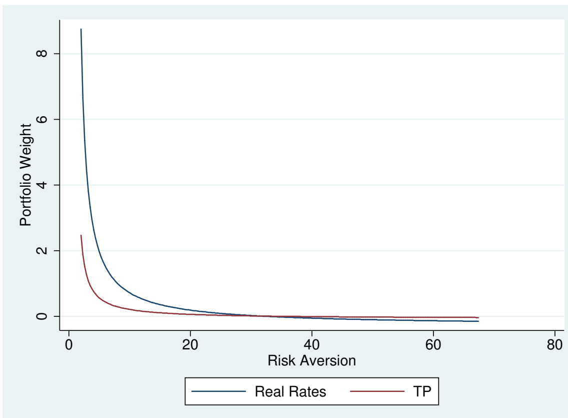


(c) Liabilities III

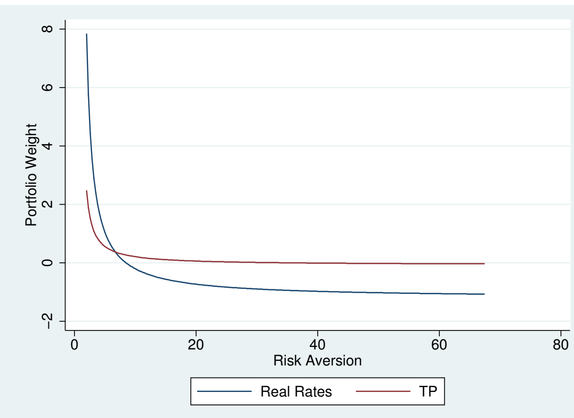


(d) Liabilities IV

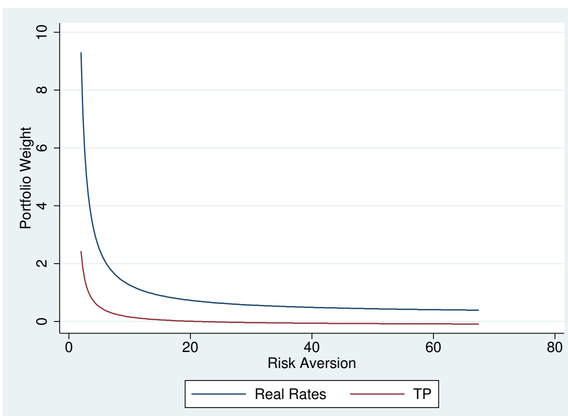
Figure 10. : Optimal Factor Allocation / Macro Model III by Liabilities Portfolio



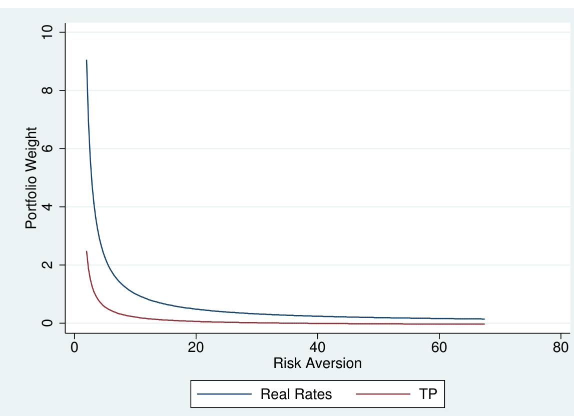
(a) Liabilities I



(b) Liabilities II

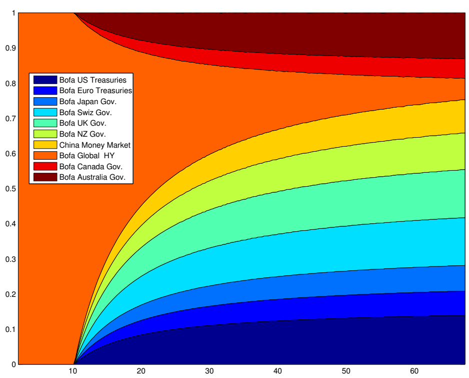


(c) Liabilities III

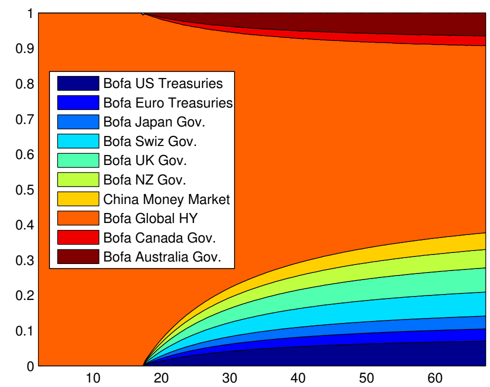


(d) Liabilities IV

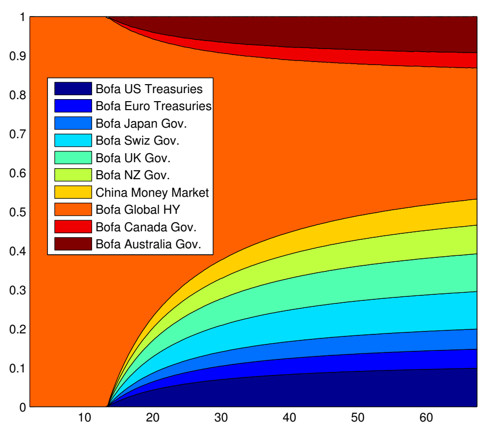
Figure 11. : Optimal Factor Allocation / Macro Model IV by Liabilities Portfolio



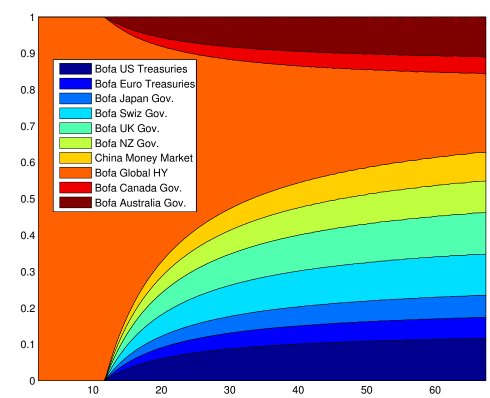
(a) Liabilities I



(b) Liabilities II

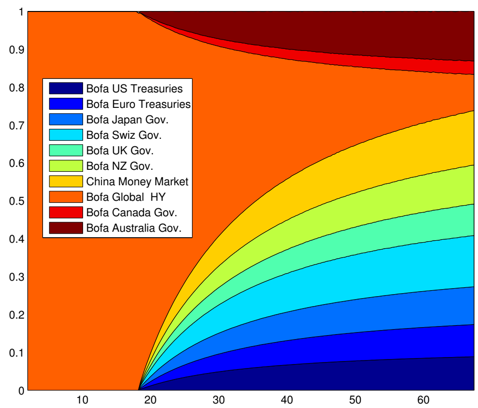


(c) Liabilities III

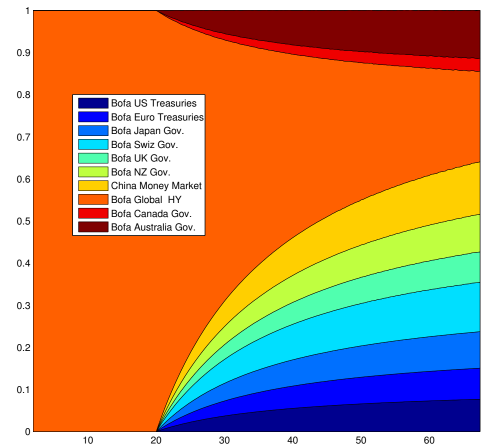


(d) Liabilities IV

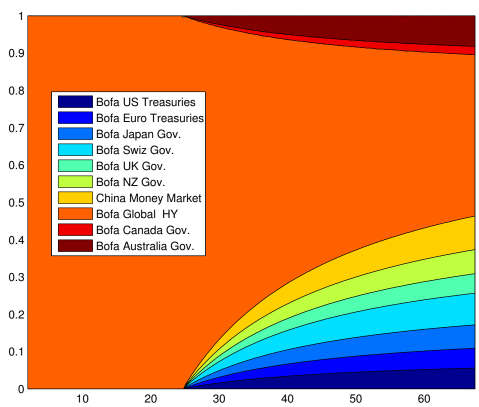
Figure 12. : Replicating Factor Portfolio / Fama-French 3 Factor Model / Fixed Income Set



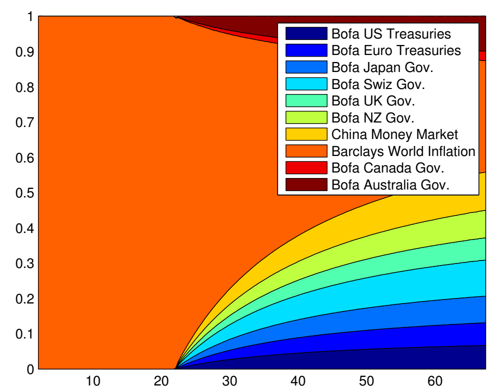
(a) Liabilities I



(b) Liabilities II

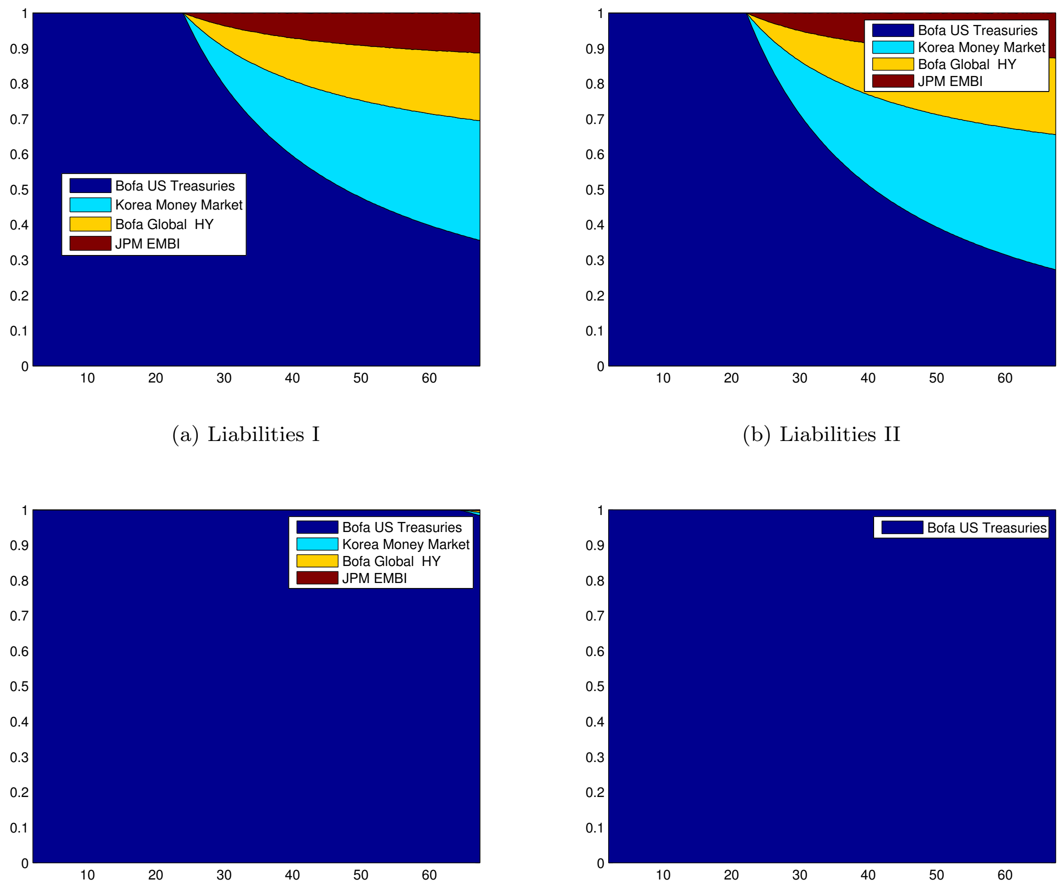


(c) Liabilities III



(d) Liabilities IV

Figure 13. : Replicating Factor Portfolio / Fama-French 5 Factor Model / Fixed Income Set



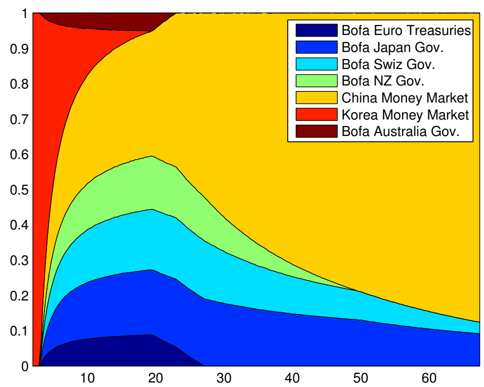
(a) Liabilities I

(b) Liabilities II

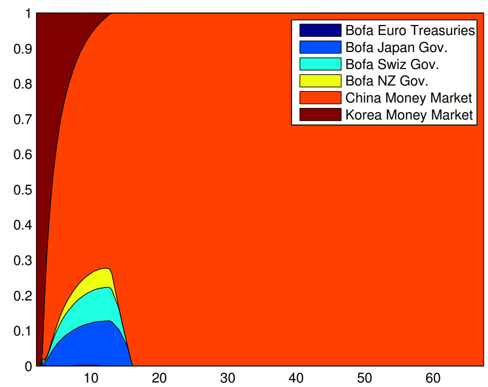
(c) Liabilities III

(d) Liabilities IV

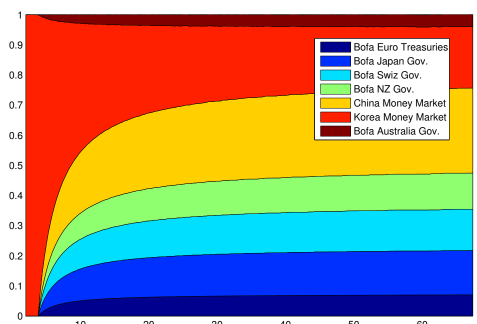
Figure 14. : Replicating Factor Portfolio / Macro Model I / Fixed Income Set



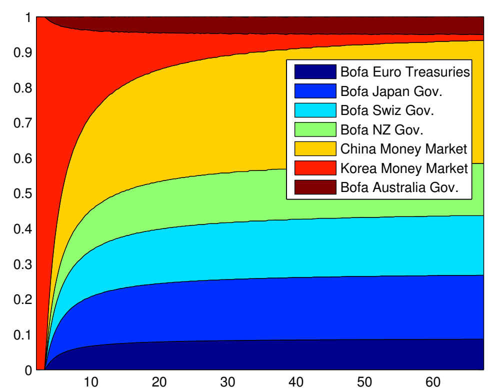
(a) Liabilities I



(b) Liabilities II

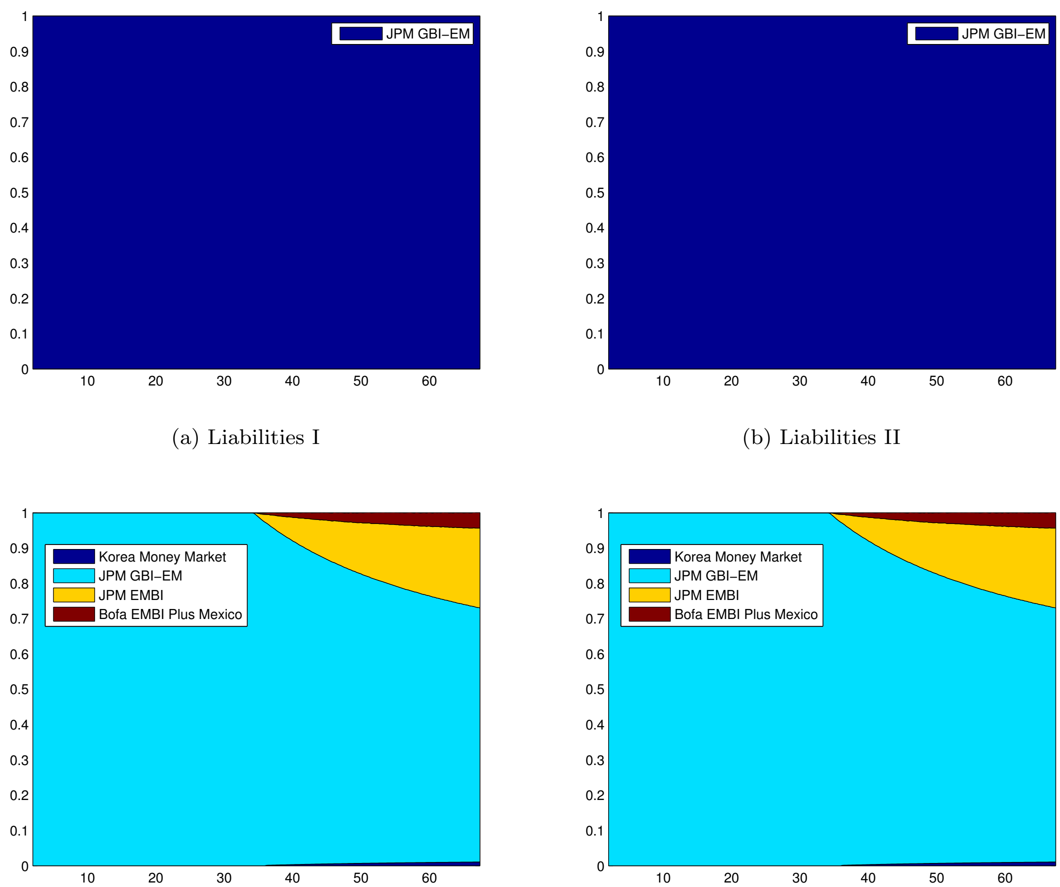


(c) Liabilities III



(d) Liabilities IV

Figure 15. : Replicating Factor Portfolio / Macro Model II / Fixed Income Set



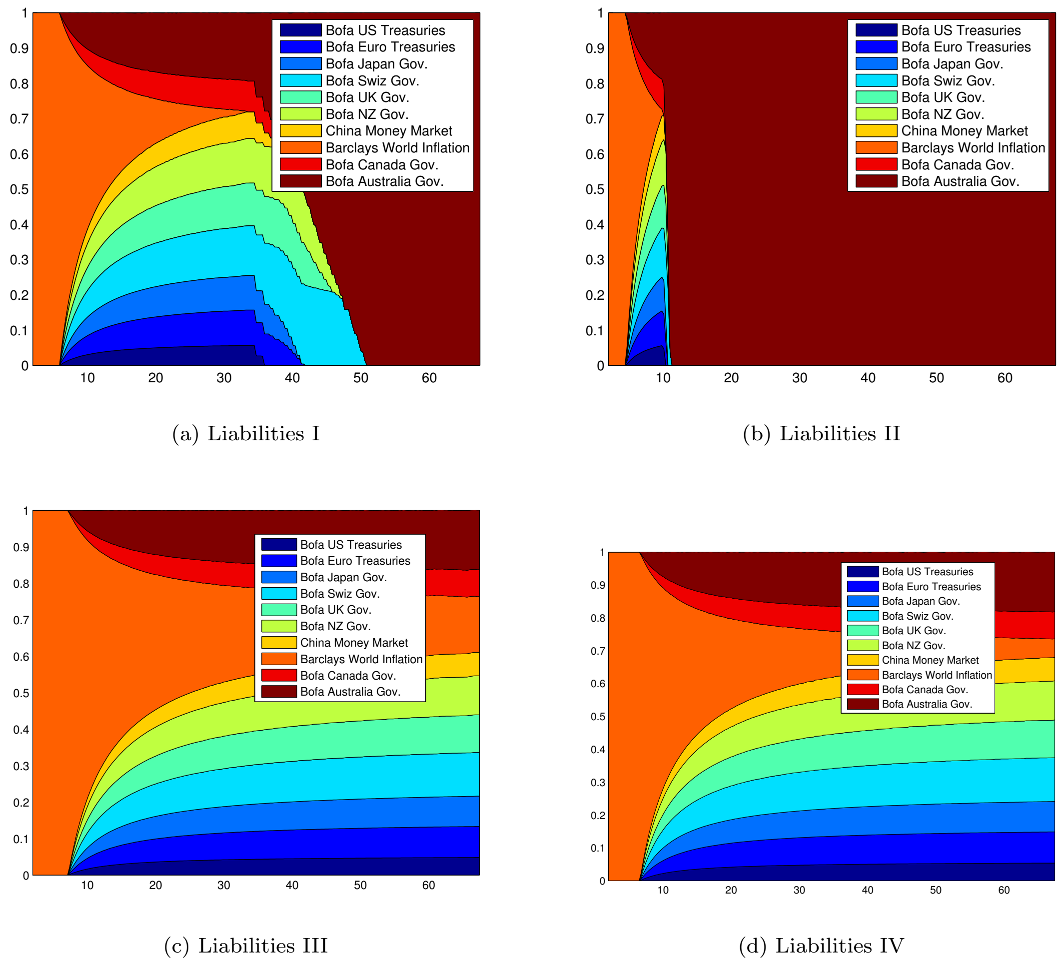
(a) Liabilities I

(b) Liabilities II

(c) Liabilities III

(d) Liabilities IV

Figure 16. : Replicating Factor Portfolio / Macro Model III / Fixed Income Set



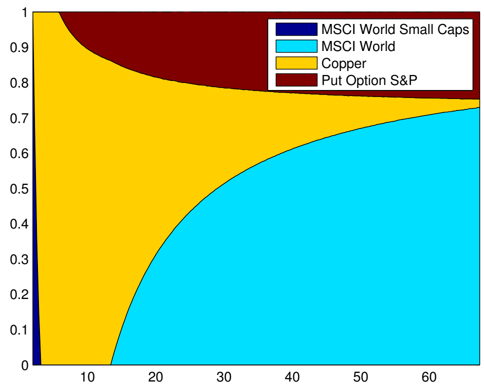
(a) Liabilities I

(b) Liabilities II

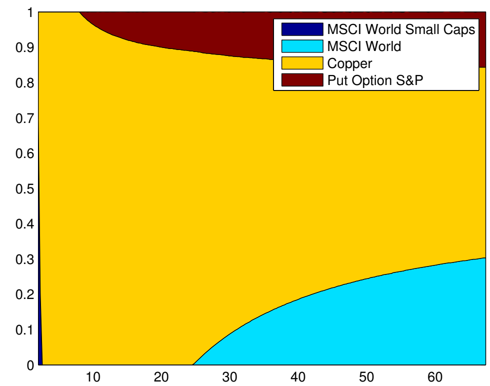
(c) Liabilities III

(d) Liabilities IV

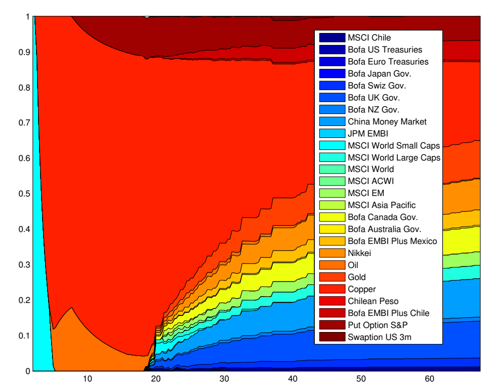
Figure 17. : Replicating Factor Portfolio / Macro Model IV / Fixed Income Set



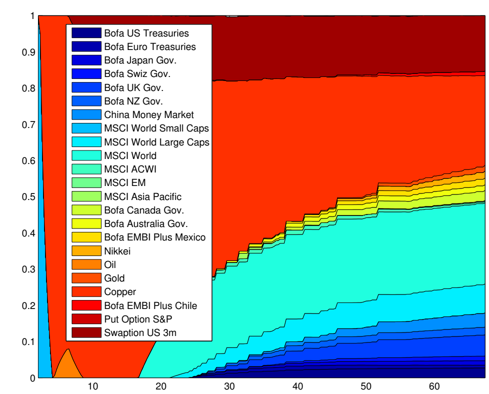
(a) Liabilities I



(b) Liabilities II

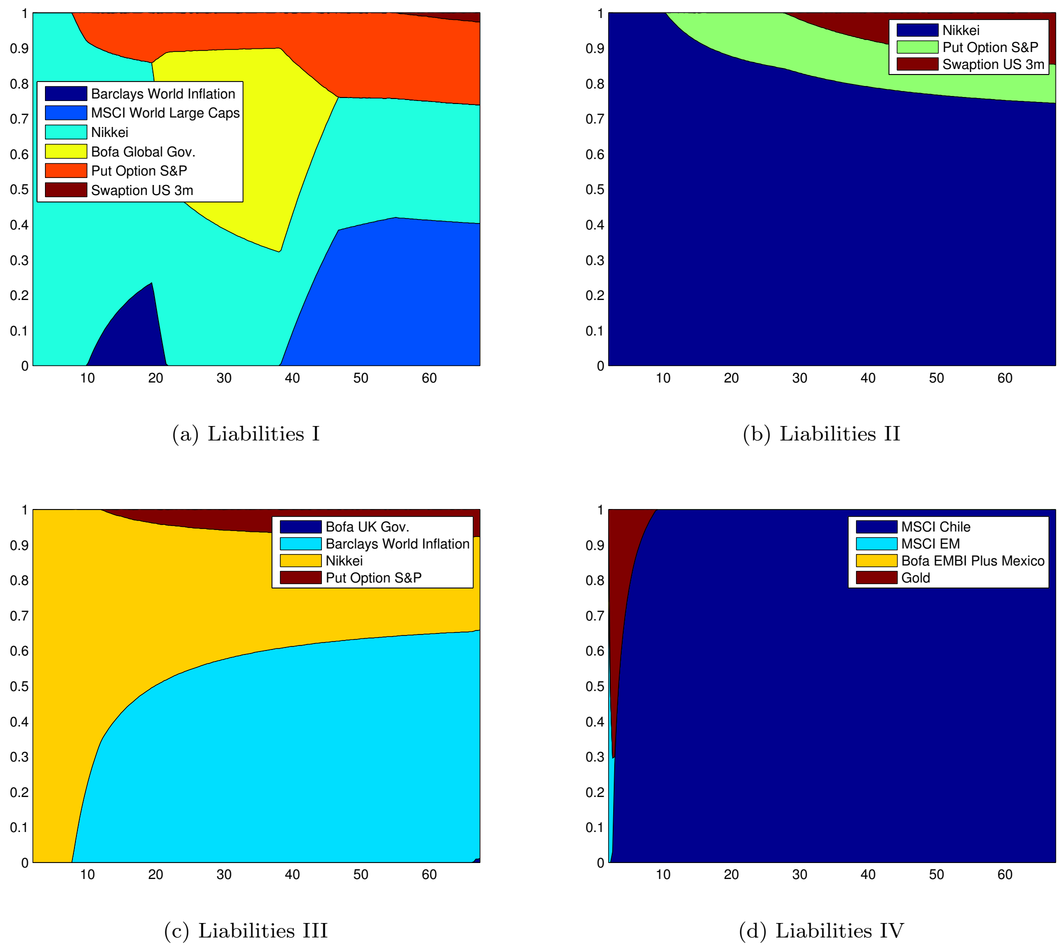


(c) Liabilities III



(d) Liabilities IV

Figure 18. : Replicating Factor Portfolio / Fama-French 3 Factor Model / All Assets



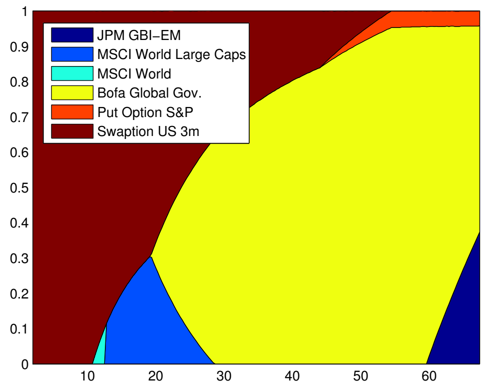
(a) Liabilities I

(b) Liabilities II

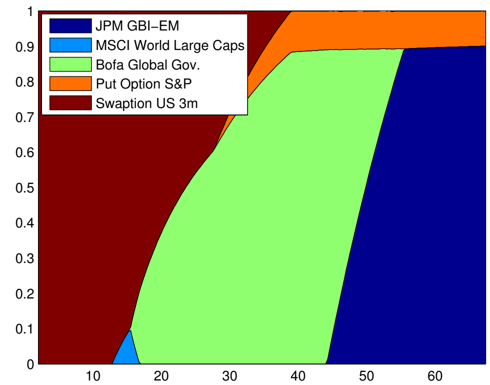
(c) Liabilities III

(d) Liabilities IV

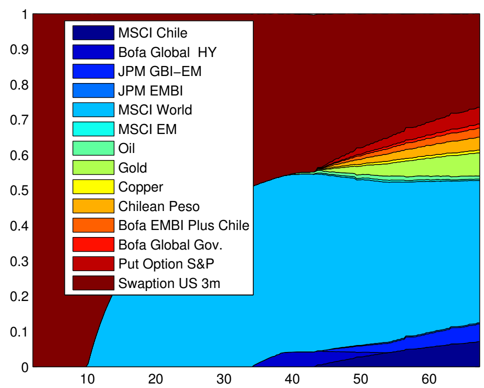
Figure 19. : Replicating Factor Portfolio / Fama-French 5 Factor Model / All Assets



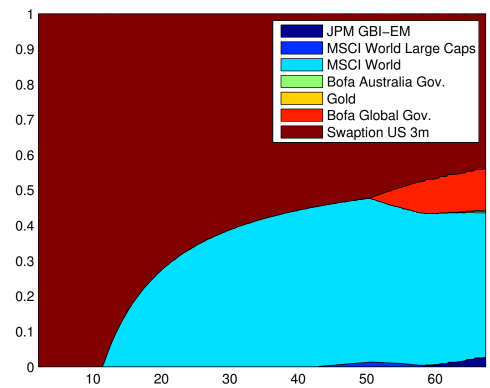
(a) Liabilities I



(b) Liabilities II

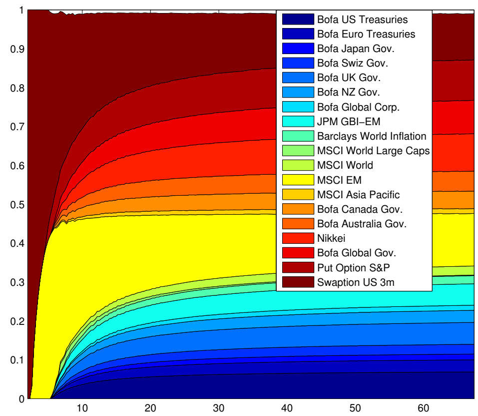


(c) Liabilities III

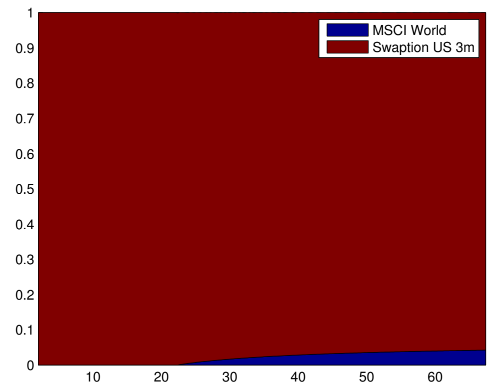


(d) Liabilities IV

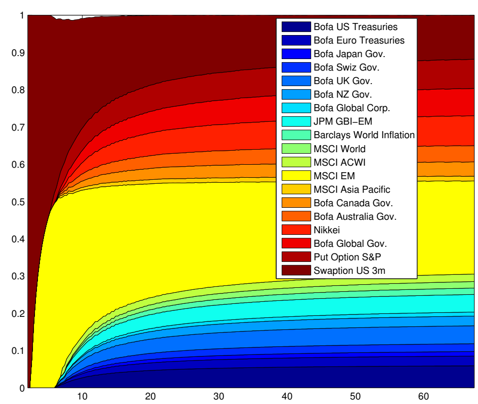
Figure 20. : Replicating Factor Portfolio / Macro Model I / All Assets



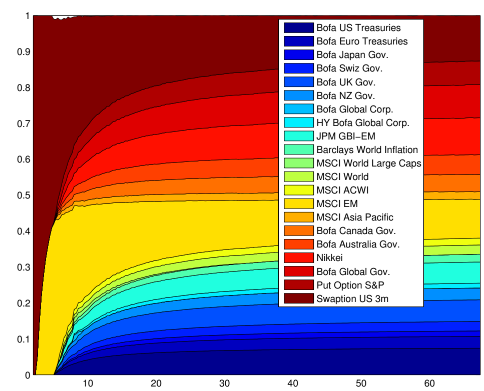
(a) Liabilities I



(b) Liabilities II

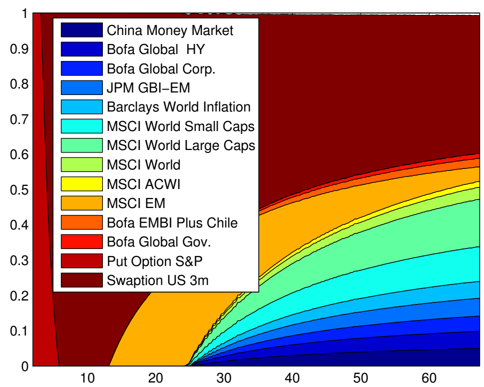


(c) Liabilities III

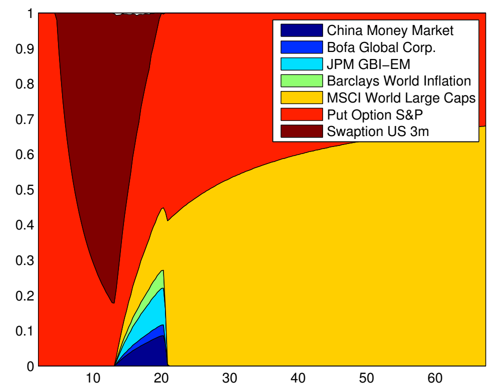


(d) Liabilities IV

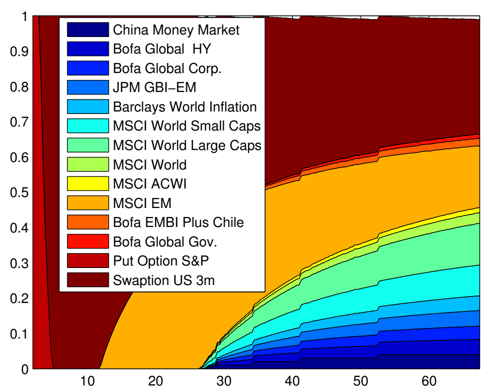
Figure 21. : Replicating Factor Portfolio / Macro Model II / All Assets



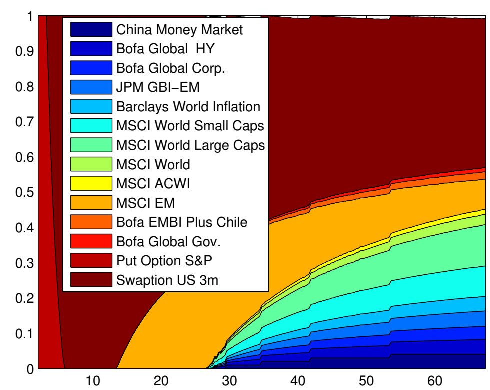
(a) Liabilities I



(b) Liabilities II

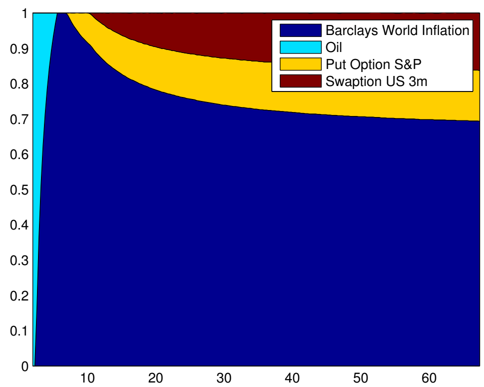


(c) Liabilities III

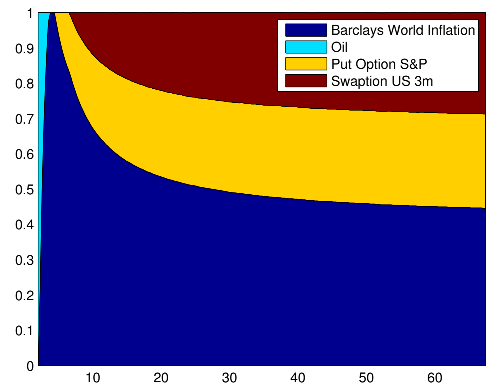


(d) Liabilities IV

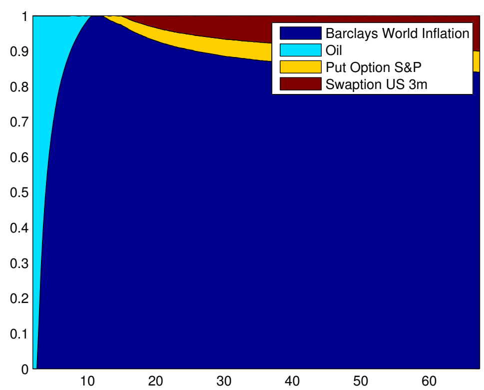
Figure 22. : Replicating Factor Portfolio / Macro Model III / All Assets



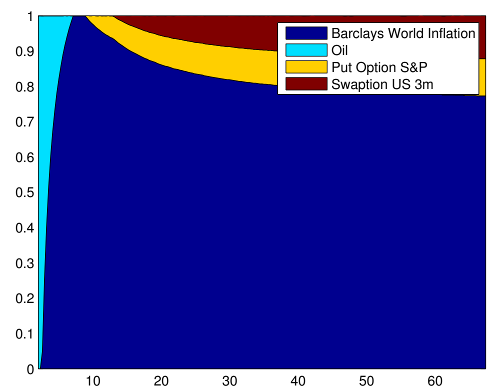
(a) Liabilities I



(b) Liabilities II

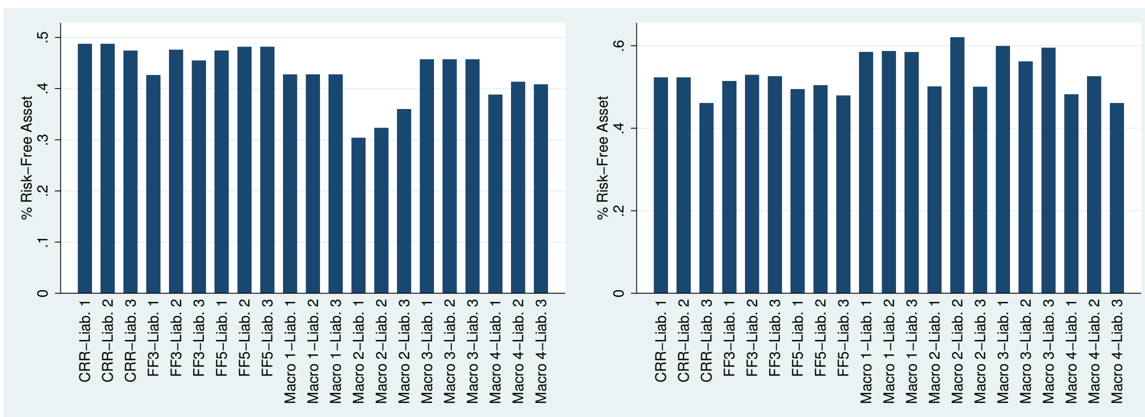


(c) Liabilities III



(d) Liabilities IV

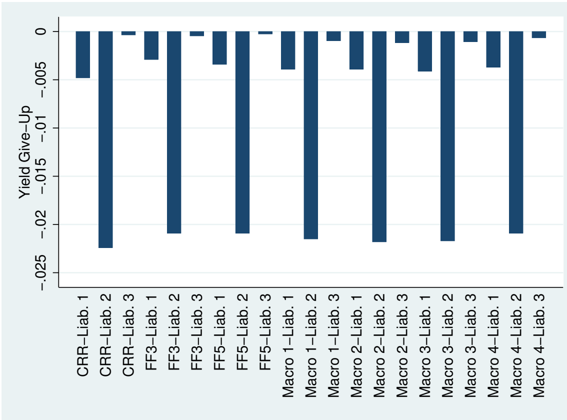
Figure 23. : Replicating Factor Portfolio / Macro Model IV / All Assets



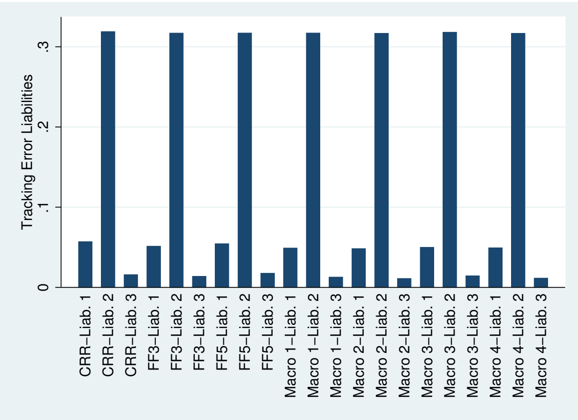
(a) % Invested in the Risk-Free Asset - Fixed Income Strategies

(b) % Invested in the Risk-Free Asset - All Assets

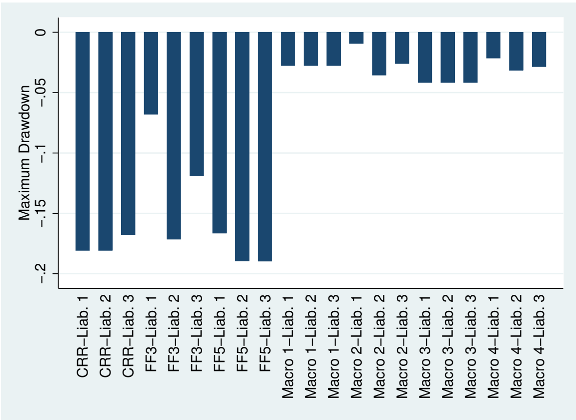
Figure 24. : Protective Put - % Invested in the Risk-Free Asset



(a) Yield Give-Up

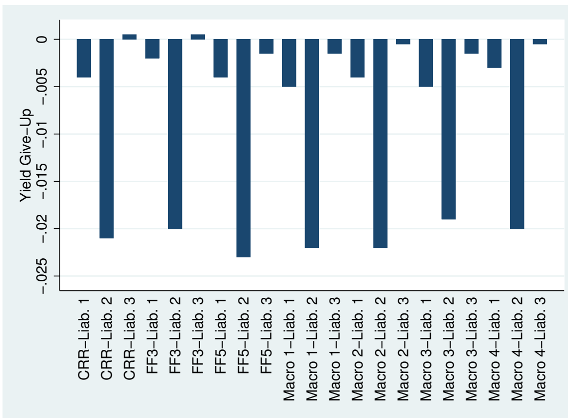


(b) Tracking Error of Liabilities

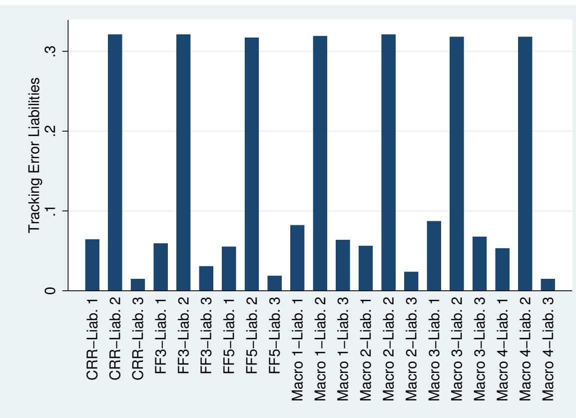


(c) Maximum Drawdown

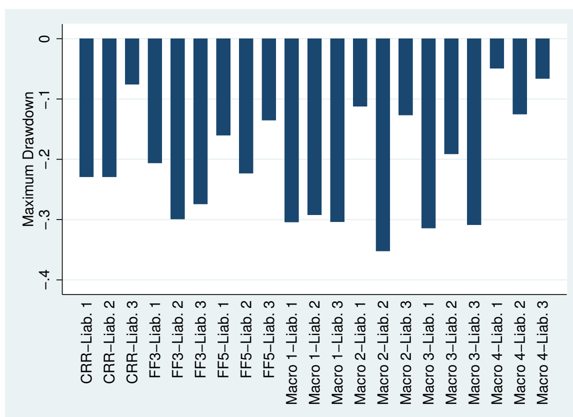
Figure 25. : Replicating Portfolio Metrics - Fixed Income Strategies



(a) Yield Give-Up

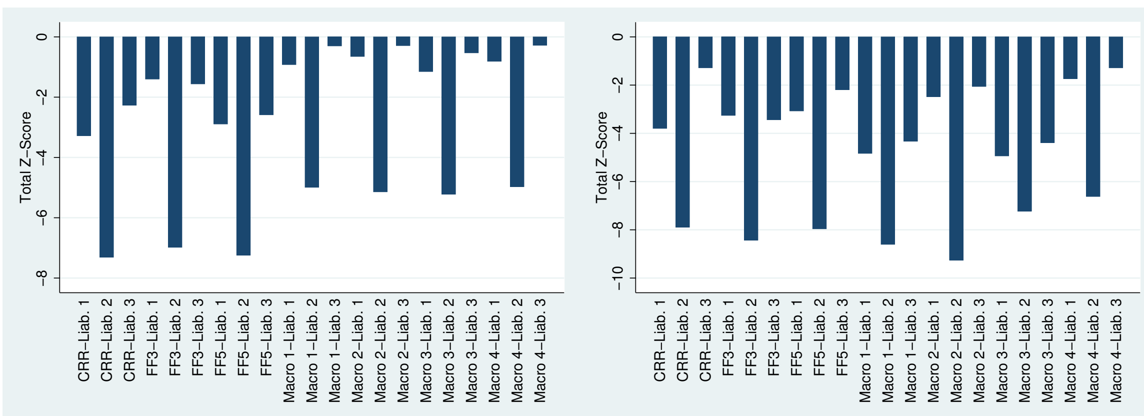


(b) Tracking Error of Liabilities



(c) Maximum Drawdown

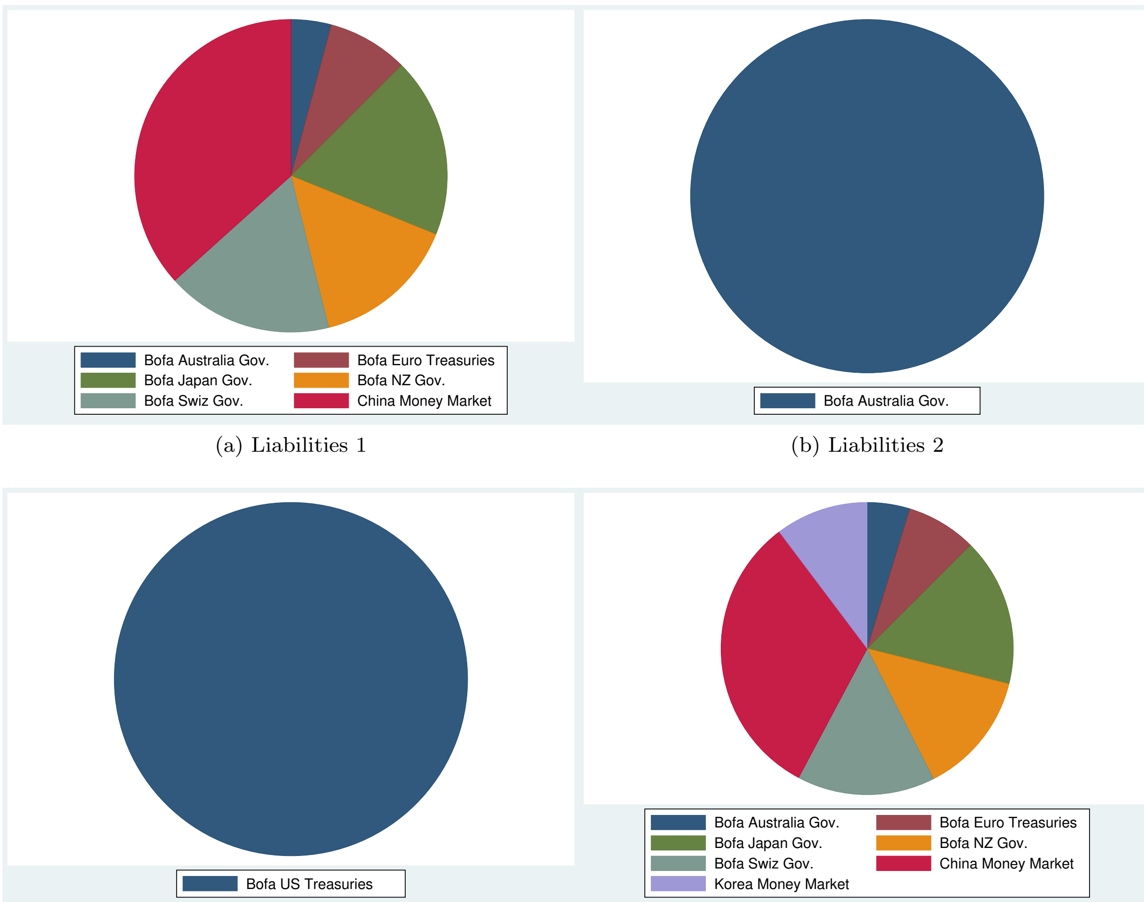
Figure 26. : Replicating Portfolio Metrics - All Assets



(a) Z-Score Fixed Income Strategies

(b) Z-Score All Assets

Figure 27. : Total Z-Score by Asset Spectrum



(a) Liabilities 1

(b) Liabilities 2

(c) Liabilities 3

(d) Liabilities 4

Figure 28. : Best Replicating Portfolio by Z-Score Rule - Fixed Income Strategies

(a) Liabilities 1

(b) Liabilities 2

(c) Liabilities 3

(d) Liabilities 4

Figure 29. : Best Replicating Portfolio by Z-Score Rule - All Assets